THE UNIVERSITY OF ALBERTA

COMPARATIVE STUDY OF SLAB-BEAM SYSTEMS

by

© JANKO MISIC

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

EDMONTON, ALBERTA

FALL, 1970

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled COMPARATIVE STUDY OF SLAB-BEAM SYSTEMS submitted by Janko Misic in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

The two design methods for reinforced concrete slabs proposed by ACI 318-71: Direct Design Method and Equivalent Frame Analysis, are compared with an elastic analysis performed by the method of finite differences. The variables considered include panel location, column and slab geometry, and the stiffnesses of the slab and supported elements. A procedure for performing calculations in a systematic manner for the Equivalent Frame Analysis is developed.

The results of this study show that, in general, there was reasonable agreement for design purposes between the proposed methods and the elastic solution. However, in certain instances, substantial differences were observed and in these cases an alternate procedure, which is more general in application, is suggested.

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NOMENCLATURE

c_1 = size of rectangular column in direction moments are being considered.

c_2 = size of rectangular column measured transverse to direction moments are being considered.

C = $\Sigma(1-0.63x/y)x^3y/3$, cross-sectional constant to define the torsional properties of edge beams and attached torsional members.

D = $\frac{E_s I_s}{(1-\mu)^2}$, flexural rigidity of slab.

E = modulus of elasticity.

E_b, E_c, E_s = modulus of elasticity for beam, column and slab concrete, respectively.

G = $\frac{E}{2(1+\mu)}$, shear modulus of elasticity.

h = height of column, center to center of floor or roof.

h_1, h_2 = distance between finite difference grid in x and y direction, respectively.

H = $\frac{E_b I_b}{D}$ ratio of beam flexural stiffness to slab stiffness.

H_1 = H in ℓ_1 direction.

H_2 = H in ℓ_2 direction.

Ib = moment of inertia about centroidal axis of gross-section of a beam.

Ic = moment of inertia of gross cross-section of column.

I_s = $\frac{t^3 \ell_1}{12}$, moment of inertia about centroidal axis of gross section of slab

K = $\Sigma \frac{K_c}{D}$, a measure of total flexural stiffness of a column.

K_b = flexural stiffness of a beam; moment per unit rotation.

K_c = flexural stiffness of a column; moment per unit rotation.

K_{ec} = flexural stiffness of an equivalent column as defined in section 13.4.1.5. of ACI 318-71.

K_s = flexural stiffness of a slab; moment per unit rotation.

k_t = torsional stiffness of torsional member; moment per unit rotation.

K' = $\Sigma K_c / \Sigma(K_s + K_b)$, ratio of flexural stiffness of the columns above and below the slab to the combined stiffness of the slab and beam at a joint taken in the direction moments are being determined.

K_e' = K' for exterior columns in a direction perpendicular to the edge of the system.

ℓ_c = length of clear span, in the direction moments are being considered measured face-to-face of supports.

ℓ_1 = length of span in the direction moments are being considered, measured center-to-center of supports.

ℓ_2 = length of span transverse to ℓ_1, measured center-to-center of supports.

m_x, m_y = bending moments per unit width of slab in x and y direction, respectively.

m_{xy}, m_{yx} = twisting moments per unit width of slab in y and x direction, respectively.

$\bar{m}_x$, $\bar{m}_y$ = bending moments acting on a section of slab width h_x and h_y, respectively.

$\bar{m}_{xy}$, $\bar{m}_{yx}$ = twisting moments acting on a section of slab width h_1 and h_2, respectively.

Mo = total static design moment

R = $E_b C/(2E_s I_s)$, ratio of torsional stiffness of edge beam section to the flexural stiffness of a width of slab equal to the span length of the beam, center to center of supports.

Ra = ratio of length of continuous edges to total perimeter of a slab panel.

t = over-all thickness of the slab.

q = design load per unit area.

w = deflection of the slab, positive downward.

W = total load on a panel.

S = ℓ_2/ℓ_1 panel aspect ratio.

x, y = shorter and longer, over-all dimension of a rectangular part of a cross-section, respectively.

x, y, z = rectangular reference coordinates.

μ = Poisson's ratio.

CHAPTER I

INTRODUCTION

1.1 Introductory Remarks

The current edition of the "Building Code Requirements for Reinforced Concrete (ACI 318-63)"[(3)*] divides all reinforced concrete slabs into two categories, namely, the two-way slabs in which each panel is supported by beams on all sides of the panel and flat slabs which are supported directly by columns with or without column capitals, and drop panels. The requirements for the design of these two categories are completely different so much so that they are presented in different chapters of the Code. This difference stems from the different origins of each procedure and the underlying philosophy of design of each. While each type of slab had its own area of use, experience and laboratory tests indicated that there was a substantial difference in the factor of safety at both, working and ultimate loads. Also the current Code does not contain provisions for considering slabs supported on beams with a complete spectrum of stiffnesses.

* Numbers in parentheses refer to entries in the list of references.

The proposed revisions of this Code, ACI 318-71[5] are based on the philosophy that both categories of slabs (that is, slabs with and without beams) should be designed by a common procedure which gives economical design with a uniform factor of safety. This philosophy is to obtain a total moment from equations of equilibrium, to split this moment between negative and positive regions, and then to proportion these moments between middle and column strips based on stiffnesses of the supporting columns and beams, if any.

For slabs containing panels that are relatively equal in size and regularly spaced, the application of this philosophy can be simplified as is done in the proposed Direct Design Method in which the proportioning of the total moment is done by a series of simple equations or coefficients. For slabs containing panels that are less regular in size and spacing a more general procedure known as the Equivalent Frame Analysis is used. All slabs considered by the Code are designed by one of these two procedures.

1.2 Object and Scope of Study

The objective of this study is to examine critically the proposed procedures contained in ACI 318-71.

This was accomplished primarily by comparing bending moments obtained by the two Code methods with those from an elastic solutions. The object was to consider the range of agreement of these comparisons, particularly at the outer values of the limitations specified in the Code, and, where the agreement was considered unsatisfactory, to propose an alternate procedure. Where possible, it was intended to provide design aids to assist in performing the calculations required by the Code procedure.

A numerical procedure based on assumptions of the elastic theory of plates was developed to obtain solutions for comparison purposes. These solutions were checked for accuracy by comparing with theoretical solutions and equations of static equilibrium. The scope of the study also includes using these solutions in order to obtain better insight into the slab-beam interaction problem.

CHAPTER II

SUMMARY OF PROPOSED REVISIONS

2.1 Introduction

In ACI 318-63[3] there were two procedures for designing flat slabs, the Empirical Method and Frame Method and three methods for designing two-way slabs which involved different sets of coefficients. In the proposed revisions, ACI 318-71[5] these procedures have been replaced by only two procedures, the Direct Design Method and Equivalent Frame Analysis.

Although the Direct Design Method is similar to previous Empirical Method in that both compute a total moment for a panel, the procedures for distributing this moment, the range of applicability and the basic philosophy are different. There is more similarity between the proposed Equivalent Frame Analysis and the previous Frame Method for flat slabs, the basic difference being in details of stiffness evaluation. The basic features of these procedures are discussed in the following sections.

2.2 Range of Applicability and Definition of Terms

Essentially, the Direct Design Method is a

modified version of the Empirical Method for flat slabs. Its range of applicability has been changed as follows: the panel length to panel width ratio and the span to successive span ratio are widened from 1.33 to 2.0 and 1.20 to 1.33, respectively. Furthermore, the definition of the column strip has been changed to be one-half the shorter span in both directions of a rectangular panel.

The major change in the Equivalent Frame Analysis is in the definition of an equivalent column. The flexibility of this equivalent column is defined as the sum of the flexibilities of the column in flexure and the beam-slab combination in the transverse direction, in torsion; i.e.

$$1/Kec = (1/K_c) + (1/K_t) \tag{2.1}$$

This change was necessitated by the recognized fact that moments "leak" from one panel to another around the column when adjacent spans are unequal in length or support different loading. The above procedure also gives more realistic moments in the exterior panel, particularly in exterior negative region.

2.3 Concept of Minimum Thickness

For two-way construction designed in accordance with the proposed ACI 318-71 the minimum thickness is

specified in section 9.5.3.1. as follows:

$$t = \frac{\ell(800 + 0.005_{fy})}{36000 + 5000\ S[H_{av} - 0.5(1-R_a)(1+\frac{1}{S})]} \qquad (2.2)$$

but not less than

$$t = \frac{\ell(800 + 0.005_{fy})}{36000 + 5000\ S(1+R_a)} \qquad (2.3)$$

The thickness need not be more than

$$t = \frac{\ell(800 + 0.005_{fy})}{36000} \qquad (2.4)$$

However, the thickness shall not be less than the following values:

For slabs without beams or drop panels	5 in.
For slabs without beams but with drop panels	4 in.
For slabs having beams on all four edges with a value of H_{av} at least equal to 2.0	3½ in.

It would appear that determination of slab thickness is somewhat more complicated in the proposed Code than existed in ACI 318-63 edition. However, it should be noted that proposed procedure considers major variables affecting stiffness and deflections.

The effect of each variable on thickness determination is shown in Fig. 2.1. To satisfy moment and shear requirements the minimum thickness is in direct proportion to the span length ℓ_1; thus expression can be considered as specifying minimum t/ℓ ratios and is so plotted. The increase of thickness for the increase of reinforcement yield stress is obtained by the coefficient k. It can be seen that the thickness decreases with increasing continuity factor, R_a. A significant factor is H_{av}, the average value of the ratios of flexural stiffness of the beam to the flexural stiffness of the slab of panel width, for all beams supporting a panel. However, for values of H_{av} greater than 2.0, the lower limit governs for all values of the continuity factor, R_a.

2.4 Concept of Total Moment

In 1914 Nichols stated that the total moment in a panel for a flat plate can be obtained by the equations of statics. His expression of the total static design moment for the interior panel with round capital is given in Eqn.(2.5). The assumptions involved in the derivation were that the reaction is uniformly distributed around the capital and twisting moments between slab and capital are neglected.

$$Mo = 0.125W\ell_1[1 - \frac{\ell_1 c_1}{\pi \ell_1} + \frac{1}{3}(\frac{c_1}{\ell_1})^3] \qquad (2.5)$$

Nichols suggested an approximation to the above equation as follows:

$$Mo = 0.125W\ell_1(1-\frac{2}{3}\,\frac{c_1}{\ell_1})^2 \qquad (2.6)$$

Based on results of test structures, Nichols' expression for total moment was considered conservative and ACI Code requirements in 1917 permitted a total moment in square interior panels of only 72 percent for the total moment (Eqn. 2.7). This was modified in 1956 by the introduction of the factor F (Eqn. 2.8), where

$F = 1.15 - \frac{c_1}{\ell_1}$ but not less than 1.00.

$$Mo = 0.09W\ell_1(1-\frac{2}{3}\,\frac{c_1}{\ell_1})^2 \qquad (2.7)$$

$$Mo = 0.09\ FW\ell_1(1-\frac{2}{3}\,\frac{c_1}{\ell_1})^2 \qquad (2.8)$$

It should be mentioned that the reason the test results did not verify Nichols' analysis was that the moments were computed from steel strains on the basis of the straight-line formula, neglecting the effect of twisting moments at the column capitals and the effect of unloaded

spans adjacent to the loaded panel.

The total moment specified by ACI 318-71 is based on Nichols' analysis as follows:

$$Mo= \frac{w\ell_2 \ell c^2}{8} \tag{2.9}$$

This is essentially the same as the moment given by Eqn. (2.5) written in terms of clear span ℓc. While the total moment required in ACI 318-71 appears greater than previously it is compensated somewhat by the reduction of the load factors.

2.5 Distribution of Total Moment

By the Direct Design Method the total design moment is distributed to the negative and positive design regions by set coefficients for the interior panel and as a function of the column beam and slab stiffnesses for the exterior panel. By the Equivalent Frame Analysis the negative and positive design moments are found directly by using a moment distribution procedure which considers the actual applied loads.

The Code distributes these design moments obtained by either procedure into middle and column strips by a set of fuctions which are dependent, for interior

panels, on the beam flexural stiffness and panel aspect ratio and, for exterior panels, on the flexural and torsional stiffnesses of the beams and the panel aspect ratio. These fuctions are fully discussed in Chapter V.

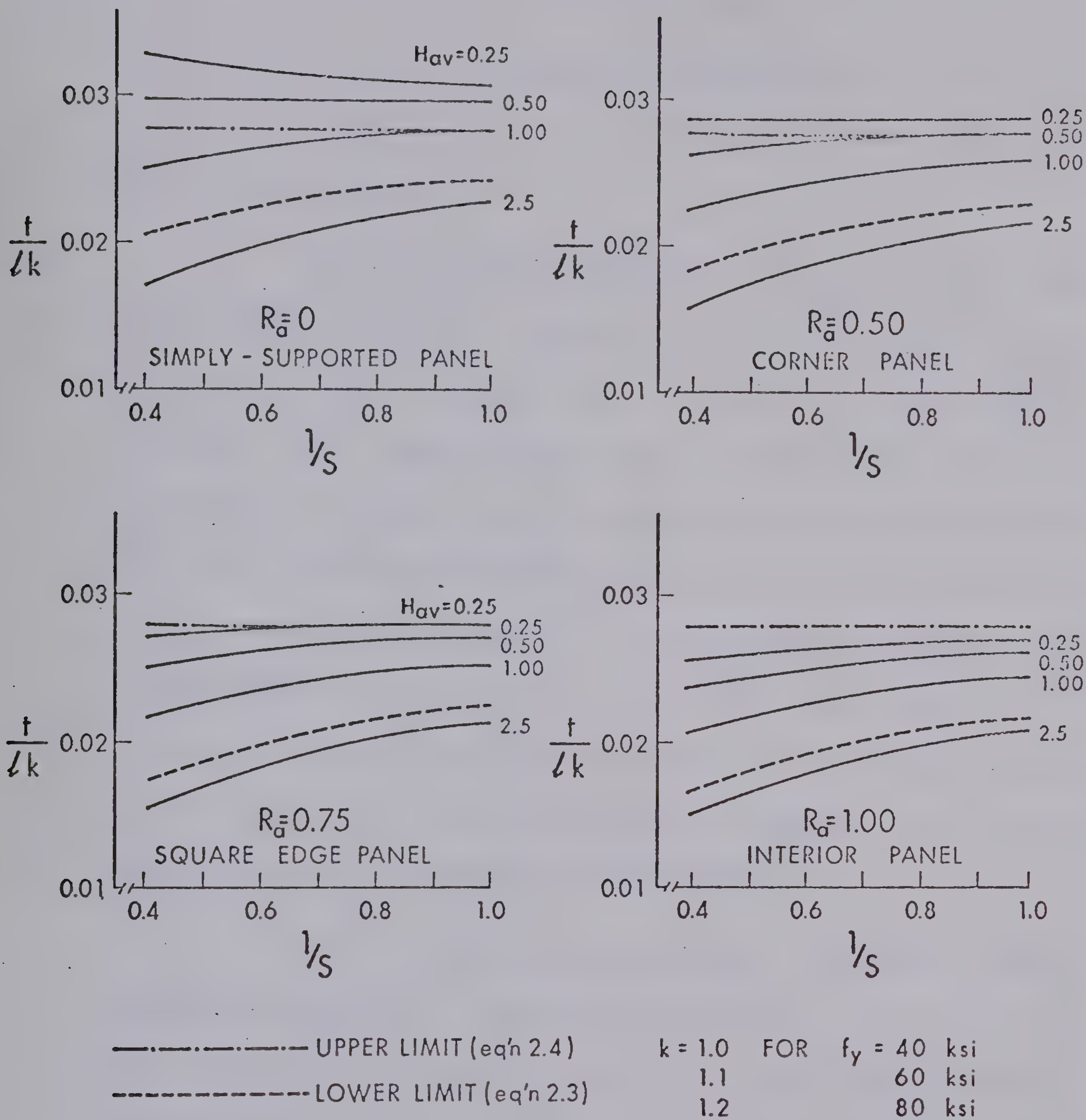


FIGURE 2.1 MINIMUM SLAB THICKNESSES

CHAPTER III

PROCEDURES OF ANALYSIS

3.1 Introduction

ACI 318-71 proposes two design methods, the Direct Design Method and the Equivalent Frame Analysis, either of which can be used for slabs with or without beams. Also, Code section 13.2.1 gives freedom to the designer, stating that the slab system may be analysed by any procedure satisfying equilibrium and geometrical compatibility, provided it is shown that strength and serviceability requirements are met. The numerical method, referred to in this study as the elastic solution meets the above requirements and therefore can be considered as a recognized design method.

In the following sections, the Direct Design Method, Equivalent Frame Analysis and elastic solution are discussed. The philosophy on which Direct Design Method is based is straight forward; hence more attention is paid to the other procedures. Design aids are presented for the Equivalent Frame Analysis in the form of tables for stiffness coefficients. A systematic procedure for performing the necessary calculations for the Equivalent Frame Analysis using these design aids was

developed and presented in detail in Appendix A.

The elastic solution is based on the finite difference equations. Details of assumptions and procedure are contained in Appendix B. The flow chart and description of the program required for the elastic solution are outlined in Appendix C. A brief description of the procedure, the types of variables that can be considered, and the accuracy of the results are presented in this chapter.

3.2 Direct Design Method

Within the limitations given in section 13.3.1 of ACI 318-71, a slab system can be analysed by the Direct Design Method. Essentially this method consists of determining the total static design moment for the panel, distributing this moment between the positive and negative regions and then proportioning these moments to column and middle strips. The effects of pattern loading are considered by providing either minimum support stiffnesses or increasing the positive resistance of the panel. In order to employ this method, evaluation of stiffness coefficients K', H and R is required which are parameters defining the column flexural stiffness, beam flexural stiffness and beam torsional stiffness, respectively. Effects of these

parameters on the design moments is shown extensively in Chapter V.

3.3 Equivalent Frame Analysis

3.3.1 Choice of Frame

The idea of the Equivalent Frame Analysis lies in reducing the three dimensional slab-beam-column system into a two dimensional structure consisting of equivalent columns and equivalent beams. The structure is then analysed for the specified loading. Longitudinal and transverse directions have to be analysed separately (Fig. 3.1). A suitable method for analysing such bents is the Moment Distribution Method. Consistent with the assumptions, the simplified Two Cycle Moment Distribution Method is often used in practice. However, a method which gives an accuracy consistent with the assumptions of the procedure and based on the slope deflection equations has been developed (Appendix A). Since this method does not require the solution of simultaneous equations, it is suitable for use with desk calculators. A general example is given in Appendix A.

3.3.2 Stiffness Evaluation

In order to analyse the equivalent frame, it is

necessary to evaluate the stiffness factor, carry-over factor and fixed-end moments for members having variable cross-sections over their lengths. For the evaluation of these constants, the moment of inertia of the slab-beam element or column at any cross-section outside of the joint may be based on the gross-section. ACI 318-71 specifies that the moment of inertia of the slab-beam from the centre of the column to the column face shall be assumed equal to the moment of inertia of the slab-beam at the column face divided by the quantity $(1-c_2/\ell_2)^2$. In computing the stiffness of the column K_c, the moment of inertia shall be assumed to be infinite from the top to the bottom of slab-beam at the joint. Knowing the moment of inertia properties of all sections of each member, any method can be applied for the evaluation of the stiffness, carry-over and fixed-end moment factors. In common use for this purpose is the Column Analogy Method. Although this method is not complex, it is rather tedious, especially in this case, because of non-prismatic members. For this reason it would be desirable to predetermine and tabulate values for the carry-over, stiffness and fixed-end moment factors. Values for these quantities, valid for the slabs without beams and with rectangular columns, are presented in Tables 3.1 to 3.4. Some other cases can be found in Ref. 13. These tables are used in the example

given in Appendix A.

3.3.3 Loading

According to ACI 318-71 the structure is to be analysed for the loads supported where they are definitely known. When the live load is variable but does not exceed three-quarters of the dead load or if the live load will always be applied to all panels, the structure may be analysed for uniform live load on all panels. If neither of these conditions are met, the maximum positive moment of a panel may be assumed to occur when three-quarters of the full design live load is on the panel and on alternate panels, and maximum negative moment may be assumed to occur when three-quarters of the full design live load is on the adjacent panels only, providing that these moments are greater than those for the entire slab loading.

3.3.4 Reduction of Negative Moment

The Equivalent Frame Analysis is based on center to center column distances and therefore negative moments obtained by this procedure are column centerline moments. Since the failure cannot occur at the column centerline, these moments need not be considered critical. While ACI 318-71 states in section 13.4.2 that column faces can be considered as a critical section, it does not suggest

any means of reduction of the centerline moment to the column face. Corley[1] has suggested the following expression for reduced moment:

$$Mn = M_{¢} - \left[\frac{3Vc}{8} - \frac{c^3 w}{16}\right] \qquad (3.1)$$

where $M_{¢}$ and Mn represent moment at column centerline and the negative design moment at the column face, respectively. The quantity V represents the total shear, as determined from the Equivalent Frame Analysis. A uniform shear distribution around a square column perimeter was the assumption for deriving Eqn. (3.1). In Ref. 11 the following expression has been suggested for reducing the moment:

$$Mn = M_{¢} - \frac{Vc}{3} \qquad (3.2)$$

It can be seen that the formula (3.2) is a simplified form of Eqn. (3.1). Table 3.5 compares the results, for an interior square panel loaded to $1k/ft^2$, obtained by these two expressions with results obtained by the Direct Design Method. It should be noted that the Direct Design Method is based on the clear span length and thus gives moments at the column face directly.

It appears that several factors have a significant influence on the reduction of negative moments, and further research in this field may be necessary.

3.3.5 Column and Middle Strip Split

The distribution of the design negative and positive moments along the design sections can be made according to the coefficients given in the Direct Design Method. Since the Code does not propose any limitations for the Equivalent Frame Analysis, it would appear that any slab can be analysed by this method. However, this is a false impression, since distribution of design moments across critical section into column and middle strip should be directly limited by section 13.3.1.6 of ACI 318-71. The necessity of applying the limitations of this section to the Equivalent Frame Analysis is discussed in Chapter V.

3.4 Elastic Solutions

3.4.1 Introduction

Elastic numerical solutions, based on the finite difference method were obtained for the purpose of evaluating the moments obtained from the Direct Design Method and Equivalent Frame Analysis. The Newmark plate analog[15], which is a physical model approach rather than mathematical

technique, was used. Therefore operators are formed from consideration of equilibrium of the elements of the model. This method is preferable when special boundary conditions are involved. In Appendix B, the procedure is outlined and characteristic operators are shown.

3.4.2 Type of Slab Analysed

A four panel typical interior strip was considered in this study (Fig. 3.3). Utilising symmetry two half-panels were solved. In the majority of the cases studied, point columns with finite stiffnesses were considered. However, a limited number of solutions with elongated columns ($c_1/\ell_1 = 0.2$) were obtained.

3.4.3 Finite Difference Grid Size and Accuracy of Results

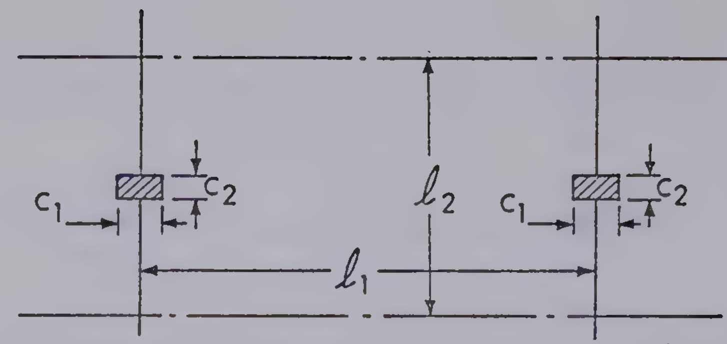
The grid spacing for both interior and exterior panels was taken as 1/20 of the panel span in each direction (Fig. 3.4). Using the symmetry about the y-axes and including fictitious points along three lines, 484 simultaneous equations were required. A Gauss Jordan elimination technique was used to solve the equations for the deflections, which in turn were used to solve for the moments and shears.

Results were obtained with an IBM 360 MOD/67 computer. The program permitted the imput of twelve variables, involving slab geometry, support stiffnesses and loading, and output the deflections, bending moments, torsion moment and shears at each grid joint.

Extremely good accuracy, based on a statics check for the total moment and total shear, was obtained. In all cases, for both interior and exterior panel moment from the analysis was exactly the same as theoretical value up to 7 significant figures. Therefore, the results obtained by the elastic analysis were considered without error for purposes of comparison. In addition, solutions for the moments and deflections were obtained for the fully fixed plate and compared to those given by Timoshenko[6], obtained by classical procedures (Table 3.6). The agreement is excellent.

TABLE 3.1

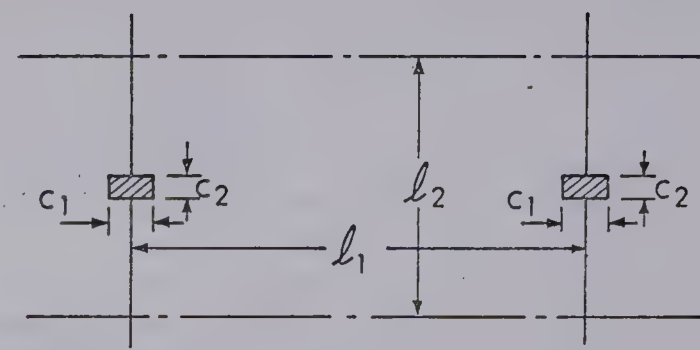
VALUES FOR CARRY-OVER FACTORS AS
DEFINED IN EQUIVALENT FRAME ANALYSIS



c_1/ℓ_1 \ c_2/ℓ_2	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500
0.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
0.050	0.500	0.503	0.507	0.510	0.513	0.516	0.518	0.521	0.523	0.526	0.528
0.100	0.500	0.506	0.513	0.519	0.524	0.530	0.535	0.540	0.545	0.550	0.554
0.150	0.500	0.509	0.517	0.526	0.534	0.543	0.550	0.558	0.565	0.572	0.579
0.200	0.500	0.511	0.522	0.532	0.543	0.554	0.564	0.574	0.584	0.593	0.602
0.250	0.500	0.512	0.525	0.538	0.550	0.563	0.576	0.588	0.600	0.612	0.623
0.300	0.500	0.514	0.527	0.542	0.556	0.571	0.585	0.600	0.614	0.628	0.642
0.350	0.500	0.514	0.529	0.545	0.560	0.576	0.593	0.609	0.626	0.642	0. 58
0.400	0.500	0.515	0.530	0.546	0.563	0.580	0.598	0.617	0.635	0.654	0.672
0.450	0.500	0.515	0.530	0.547	0.564	0.583	0.602	0.621	0.642	0.662	0.683
0.500	0.500	0.515	0.530	0.547	0.564	0.583	0.603	0.624	0.645	0.667	0.690

TABLE 3.2

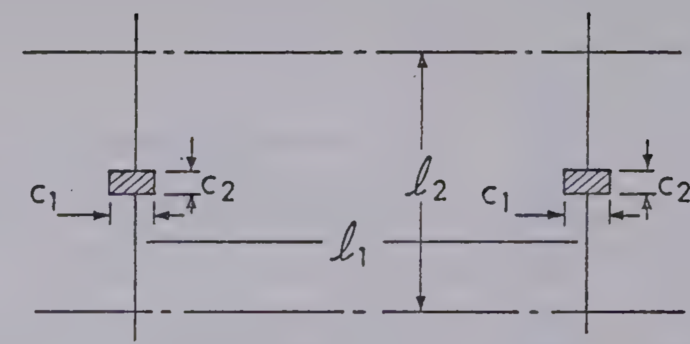
VALUES FOR THE STIFFNESS FACTORS
AS DEFINED IN EQUIVALENT FRAME ANALYSIS



c_1/ℓ_1 \ c_2/ℓ_2	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500
0.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000
0.050	4.000	4.047	4.093	4.138	4.181	4.222	4.261	4.299	4.334	4.368	4.398
0.100	4.000	4.091	4.182	4.272	4.362	4.449	4.535	4.618	4.698	4.774	4.846
0.150	4.000	4.132	4.267	4.403	4.541	4.680	4.818	4.955	5.090	5.222	5.349
0.200	4.000	4.170	4.346	4.529	4.717	4.910	5.108	5.308	5.509	5.710	5.908
0.250	4.000	4.204	4.420	4.648	4.887	5.138	5.401	5.672	5.952	6.238	6.527
0.300	4.000	4.235	4.488	4.760	5.050	5.361	5.692	6.044	6.414	6.802	7.205
0.350	4.000	4.264	4.551	4.864	5.204	5.575	5.979	6.416	6.888	7.395	7.935
0.400	4.000	4.289	4.607	4.959	5.348	5.778	6.255	6.782	7.365	8.007	8.710
0.450	4.000	4.311	4.658	5.046	5.480	5.967	6.517	7.136	7.836	8.625	9.514
0.500	4.000	4.331	4.703	5.123	5.599	6.141	6.760	7.470	8.289	9.234	10.329

TABLE 3.3

VALUES FOR THE FIXED-END MOMENTS COEFFICENTS
AS DEFINED IN EQUIVALENT FRAME ANALYSIS

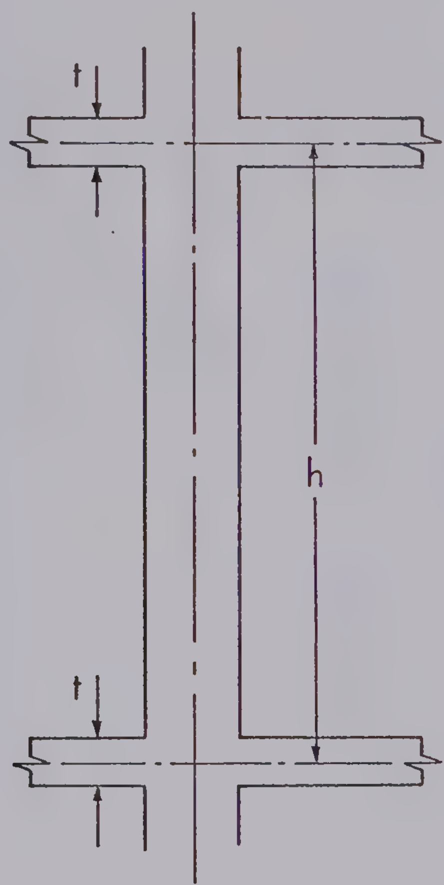


c_1/ℓ_1 \ c_2/ℓ_2	0.000	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500
0.000	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083
0.050	0.083	0.084	0.084	0.084	0.085	0.085	0.085	0.086	0.086	0.086	0.086
0.100	0.083	0.084	0.085	0.085	0.086	0.087	0.087	0.088	0.088	0.089	0.089
0.150	0.083	0.084	0.085	0.086	0.087	0.088	0.089	0.090	0.090	0.091	0.092
0.200	0.083	0.085	0.086	0.087	0.088	0.089	0.090	0.091	0.092	0.093	0.094
0.250	0.083	0.085	0.086	0.087	0.089	0.090	0.091	0.093	0.094	0.095	0.096
0.300	0.083	0.085	0.086	0.088	0.089	0.091	0.092	0.094	0.095	0.096	0.098
0.350	0.083	0.085	0.087	0.088	0.090	0.091	0.093	0.095	0.096	0.098	0.099
0.400	0.083	0.085	0.087	0.088	0.090	0.092	0.094	0.095	0.097	0.099	0.100
0.450	0.083	0.085	0.087	0.088	0.090	0.092	0.094	0.096	0.098	0.100	0.101
0.500	0.083	0.085	0.087	0.088	0.090	0.092	0.094	0.096	0.098	0.100	0.102

TABLE 3.4

VALUES OF kc, STIFFNESS COEFFICENTS OF THE COLUMNS AS DEFINED IN EQUIVALENT FRAME ANALYSIS*

t/h	kc
0.00	4.000
0.01	4.102
0.02	4.208
0.03	4.318
0.04	4.433
0.05	4.552
0.06	4.676
0.07	4.805
0.08	4.940
0.09	5.080
0.10	5.226
0.11	5.379
0.12	5.458
0.13	5.539
0.14	5.705
0.15	5.879
0.16	6.061
0.17	6.252
0.18	6.452
0.19	6.661
0.20	6.880



* Stiffness of the column is

$$K_c = k_c \frac{E_c I_c}{h}$$

TABLE 3.5

REDUCTION OF NEGATIVE MOMENT FOR AN INTERIOR SQUARE PANEL

ℓ_1(ft)	12			16			20			24		
c_1/ℓ_1	A	B	C	A	B	C	A	B	C	A	B	C
0.1	113.7	114.1	117.6	269.6	270.4	278.7	526.5	528.2	544.4	909.8	912.7	940.7
0.2	89.9	88.1	94.4	213.0	208.9	223.9	416.0	407.9	437.2	718.8	704.9	755.6
0.3	68.8	65.2	73.1	163.1	154.5	173.2	318.5	301.8	338.3	550.4	521.5	584.6
0.4	50.5	45.1	52.6	119.8	106.9	124.6	234.0	208.8	243.4	404.4	360.8	420.7
0.5	35.1	27.9	32.4	83.2	66.1	76.8	162.5	129.2	150.0	280.8	223.2	259.2

A - Direct Design Method

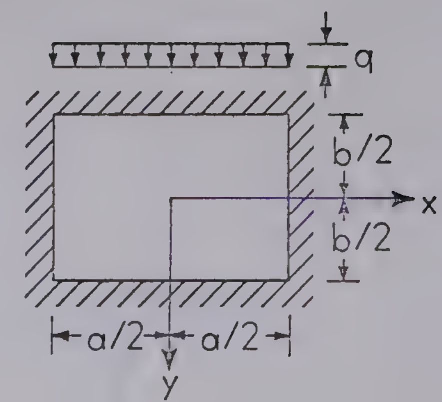
B,C - Equivalent Frame Analysis:

B - Reduced by Corley's expression $(\frac{3Vc}{8} - \frac{c^3w}{16})$

C - Reduced by simplified formula $(\frac{Vc}{3})$

TABLE 3.6

COMPARISON OF RESULTS OBTAINED BY TIMOSHENKO[6] AND ELASTIC ANALYSIS



b/a	$(w)_{x=o,\ y=o}$ (qa^4/D)		$(M_x)_{x=a/2,\ y=o}$ (qa^2)		$(M_x)_{x=o,\ y=o}$ (qa^2)	
	Timoshenko	Elastic Analysis	Timoshenko	Elastic Analysis	Timoshenko	Elastic Analysis
1.0	0.00126	0.00129	-0.0513	-0.0507	0.0231	0.0230
1.1	0.00150	0.00154	-0.0581	-0.0576	0.0264	0.0268
1.2	0.00172	0.00176	-0.0639	-0.0633	0.0299	0.0301
1.3	0.00191	0.00195	-0.0687	-0.0682	0.0327	0.0329
1.4	0.00207	0.00211	-0.0726	-0.0721	0.0349	0.0355
1.5	0.00220	0.00224	-0.0757	-0.0752	0.0368	0.0369
1.6	0.00230	0.00234	-0.0780	-0.0776	0.0381	0.0383
1.7	0.00238	0.00243	-0.0799	-0.0794	0.0392	0.0394
1.8	0.00245	0.00249	-0.0812	-0.0808	0.0401	0.0402
1.9	0.00249	0.00254	-0.0822	-0.0818	0.0407	0.0408
2.0	0.00254	0.00258	-0.0829	-0.0825	0.0412	0.0413

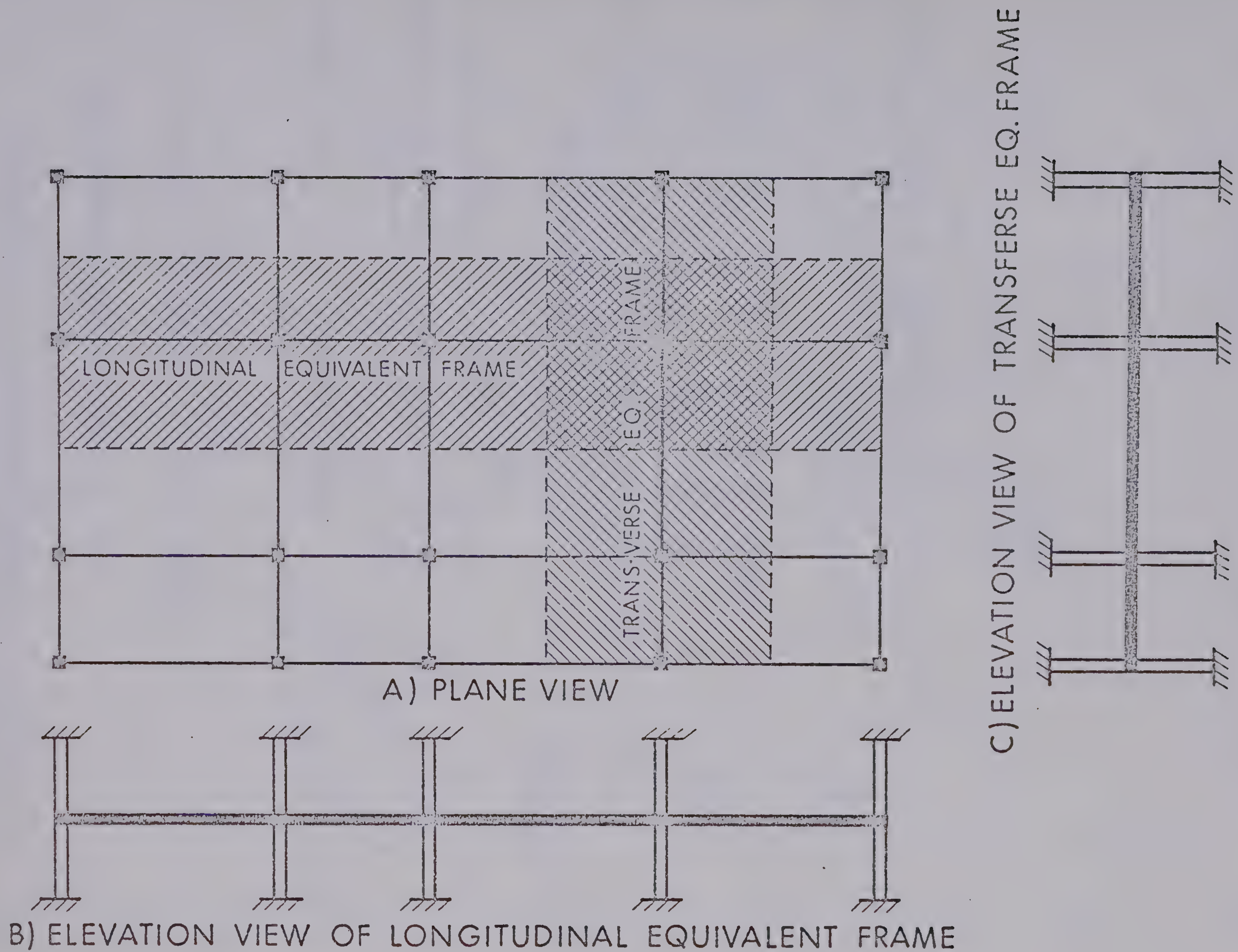


FIGURE 3.1 CHOICE OF EQUIVALENT FRAMES

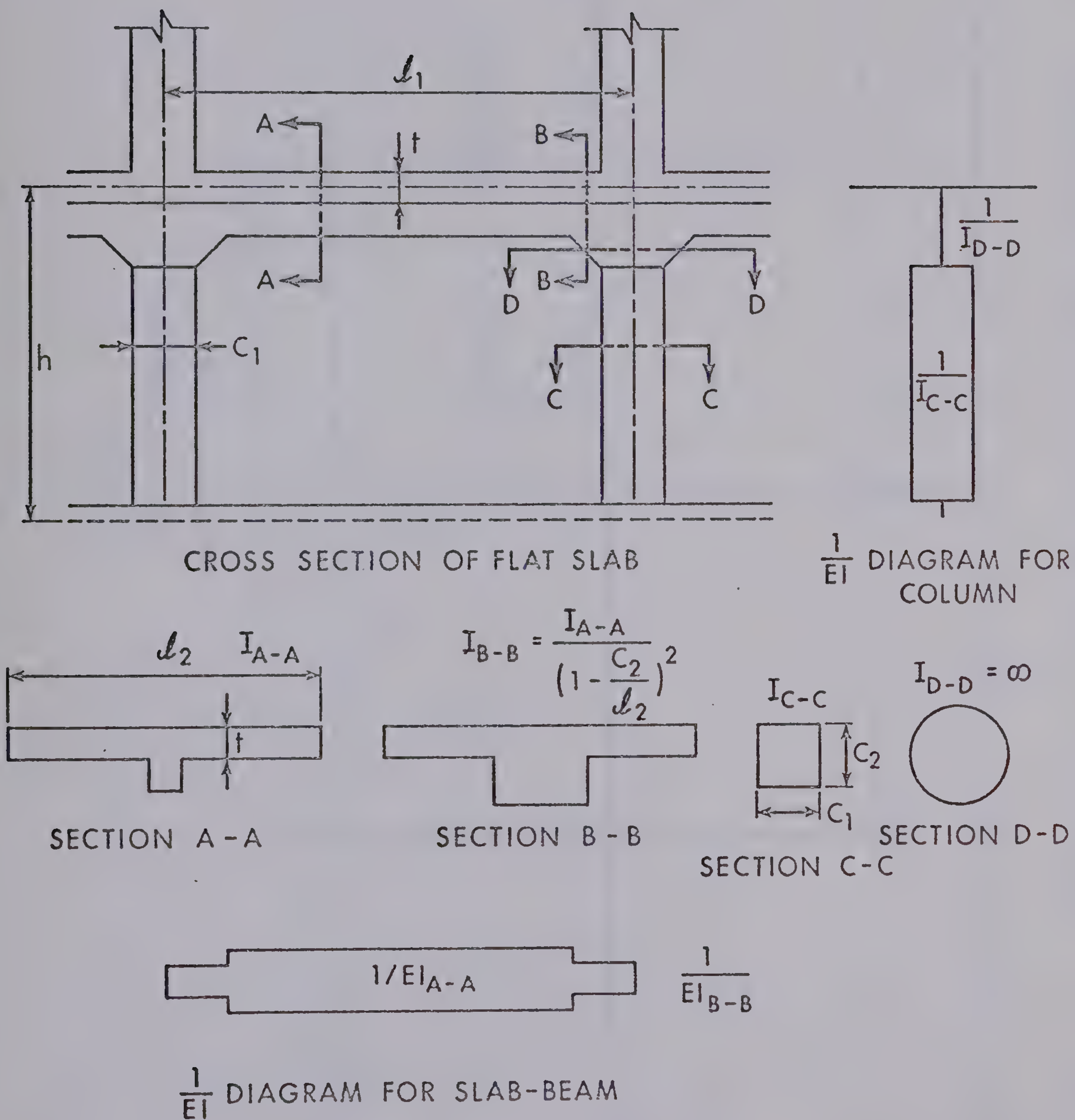


FIGURE 3.2 CROSS-SECTIONS FOR CALCULATING STIFFNESSES OF EQUIVALENT FRAME

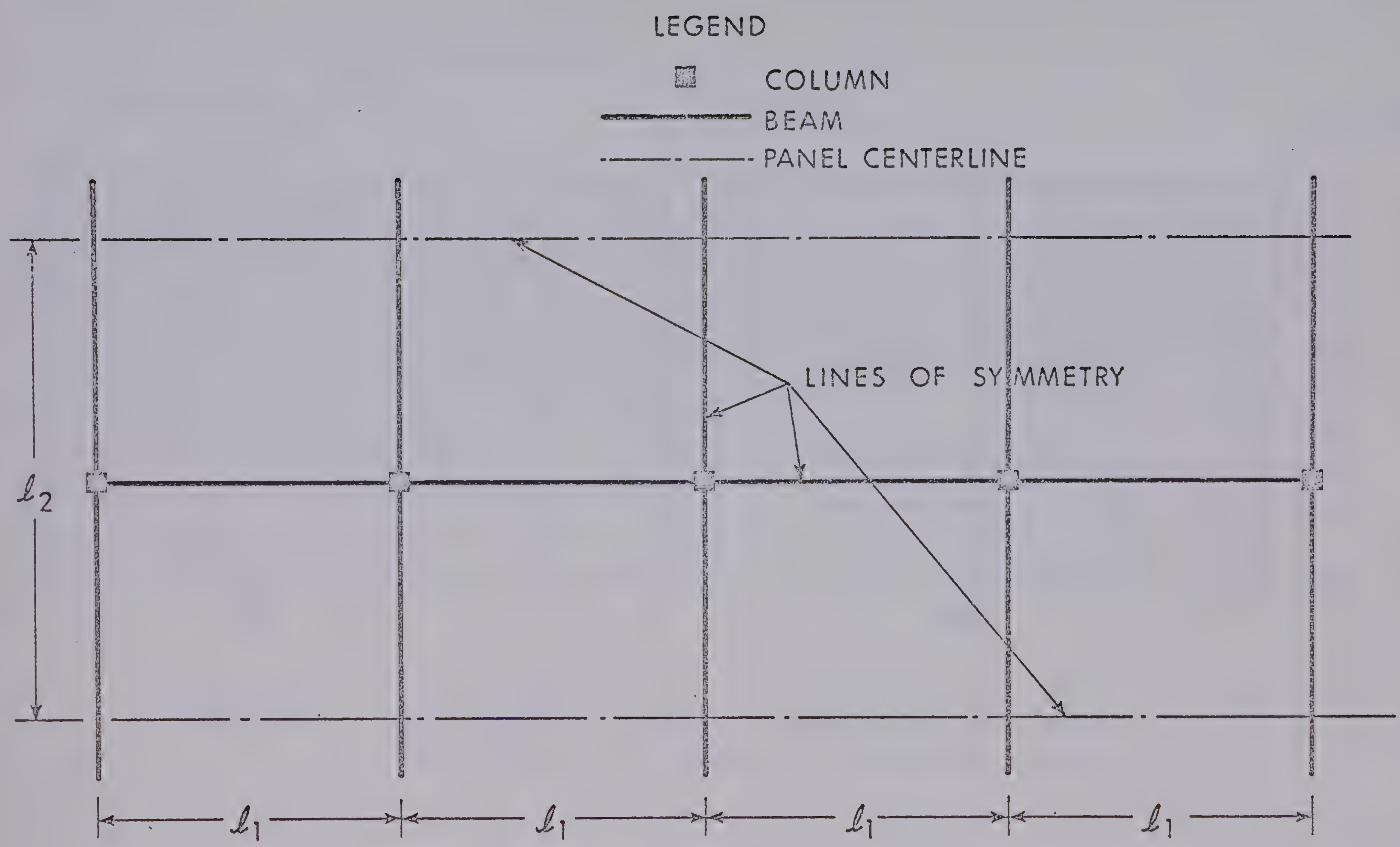


FIGURE 3.3 TYPICAL INTERIOR STRIP CONSIDERED

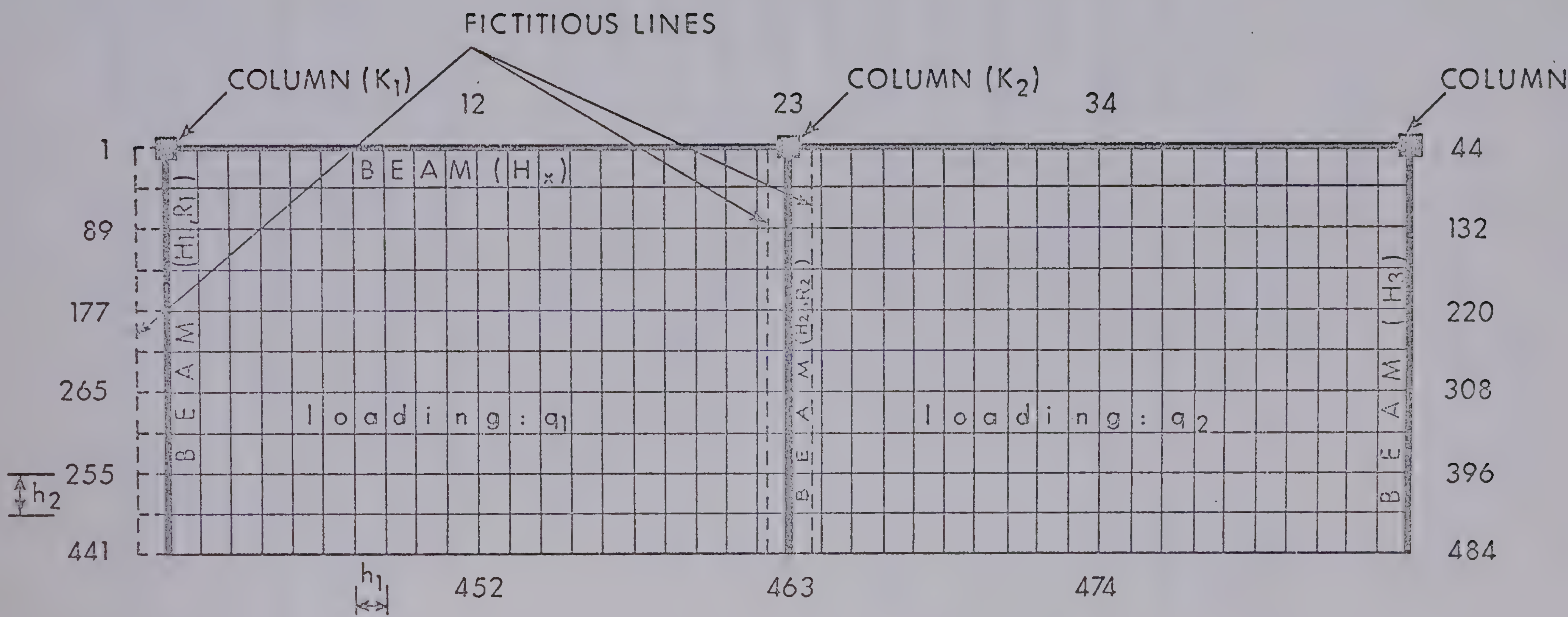


FIGURE 3.4 ARRANGEMENT OF PANELS INVESTIGATED, VARIABLES CONSIDERED, AND NUMBERING OF EQUATION SYSTEM

CHAPTER IV

PARAMETERS CONSIDERED

4.1 Introduction

The parameters used for the elastic solutions are the same as those used in the proposed revisions of the Code. In order to reduce the number of variables considered the Code has combined the parameters to form "compound variables", such as Kc/(Ks + kb), $H_1\ell_2/\ell_1$ and $H_1\ell_2{}^2/H_2\ell_1{}^2$. To establish more clearly the effect of each parameter on the slab beam interaction problem, elastic solutions were obtained using the parameters as "single variables", such as H_1, H_2, ℓ_2/ℓ_1, R, Kc, Ks and Kb. In order to facilitate a direct comparison between the elastic solution and the Code, however, elastic solutions were also obtained using the "compound variables". The discussion of stiffness variables is presented in section 4.3 in the same order as they appear in Chapter V.

4.2 Panel Location and Aspect Ratio

Since there is substantial difference in the behavior between interior and exterior panels, it was desirable to consider both in this study. This was accomplished by considering a typical interior strip consisting of four panels (Fig. 3.3). A typical interior panel was obtained by using extremely high values for the

stiffness of the exterior column and the torsional stiffness of the exterior beam. With such a stiff column and beam, the deflections of the plate on the two opposite sides of the interior column were exactly equal to sixteen decimal points; therefore, the first interior panel of the strip analysed can be considered as a typical interior for all purposes.

Whenever the panel aspect ratio was varied independently of stiffness quantities, it was taken as 0.5, 1.0, 1.5 and 2.0. In cases when the curve was not clearly defined by these four values, additional quantities were chosen.

4.3 Stiffness Parameters

For the study of splitting of the total design moment into negative and positive regions, the dimensionless factor, $1/(1 + 1/K'e)$, was varied from 0.0 to 1.0 in increments of 0.25. This meant that the exterior column stiffness parameter varied from zero to infinity. Following the Code definition, the column stiffness parameter, K', may also be written as

$$K' = \Sigma(Kc/Ks)/\Sigma(1+H)$$

By considering the numerator as one variable, the number of variables involving K' is reduced from three to two. These two variables were varied simultaneously in order to

obtain the above range of values for K'. The flexural and torsional stiffnesses of supporting beams were considered equal and taken as 0.0, 1.0 and 5.0 to represent slabs without beams and slabs supported on intermediate and stiff beams, respectively.

For the beam-slab interaction problem, the primary variables are the beam flexural stiffness in the longitudinal and transverse directions, H_1 and H_2; the panel aspect ratio, ℓ_2/ℓ_1; and in addition, for exterior panels, the torsional stiffness of the exterior beam, R. The effect of the column stiffness is slight, and for this study a column of intermediate stiffness equal to three times the slab stiffness was chosen in all solutions. For interior panels, values of H_1 corresponding to 0.0, 0.5, 1.0 and 5.0 were considered in all combinations with the same values for the ratio H_2/H_1 for each of four values of the ratio ℓ_2/ℓ_1 namely, 0.5, 1.0, 1.5 and 2.0. This required 64 solutions. For exterior panels four values of R corresponding to 0.0, 1.0, 5.0 and ∞ were combined with selected values of H_1 and H_2 for the same values of ℓ_2/ℓ_1 used for the interior panel.

For the study of slabs supported on beams in one direction only, the Code variable $H_1\ell_2/\ell_1$ or $H_2\ell_2/\ell_1$ was used as follows: 0.0, 0.5, 1.0 and 10.0.

In order to compare ACI 318-71 values with the elastic solutions, the quantity $H_1\ell_2$ ℓ_1 was varied as 0.0,

0.5, 1.0 and 10.0 while $H_1\ell_2^2/H_2\ell_1^2$ was kept constant as unity. For the exterior panel the torsional stiffness of the edge beam was considered as 0.0 and 2.5 with combinations of the above values for the parameter $H_1\ell_2/\ell_1$. Again the column stiffness was kept constant and taken as three times the stiffness of the slab. In order to examine Code limitations of variable $H_1\ell_2^2/H_2\ell_1^2$, this variable was taken to be 0.1, 0.2, 1.0, 5.0 and 10.0 for unit values of $H_1\ell_2/\ell_1$.

4.4 Loading

Although the computer program permitted the variation of the load on each panel, only a uniformly distributed load, w, over both panels was considered in this study. This load was considered as the total design load.

4.5 Poisson's Ratio

In the majority of the cases considered in this study, the value of Poisson's ratio, μ, was considered to be zero. However, for the purposes of comparison of finite difference solutions with those given by Timoshenko[6] for the plate with clamped edges, eleven solutions were obtained using Poisson's ratio equal to 0.3.

CHAPTER V

PRESENTATION AND DISCUSSION OF RESULTS

5.1 Introduction

The design moments obtained by the two ACI proposed procedures, Direct Design Method and Equivalent Frame Analysis, are compared with the results obtained by an elastic solution. The requirements for the proportioning of the total static design moment between the negative and positive regions and the splitting of these design moments into column and middle strips are critically examined within limitations given in section 13.3.1 of ACI - 318-71[5]. Special emphasis was placed on the slab-beam interaction problem and a more general procedure is suggested which can be extended to slabs which do not meet the limitations in proposed revisions.

The methods proposed in the new Code, the elastic solution, and the procedure suggested in this study are compared by considering the behavior of both interior and exterior panels for the typical interior strip. The results of the comparison are presented in the form of graphs, tables and formulas.

5.2 Total Static Design Moment

For any panel the total static design moment

depends only on the panel size, the column dimensions and the loading, and can be determined by equations of equilibrium. Fig. 5.1 shows the variation in the moment coefficient, $M_o/w\ell_1{}^3$, with respect to the aspect ratio, ℓ_2/ℓ_1, and the ratio of column size to span length, c_1/ℓ_1. Since the total static moment determined by the Direct Design Method is based on the consideration of the statics, the fact that the solution obtained by the elastic analysis agrees exactly for all values of panel and column aspect ratios, represents an excellent check for the accuracy of the elastic solution.

5.3 Negative and Positive Design Moment

5.3.1 Interior Panel

With the Direct Design Method, the design moments are based on the clear span, ℓ_c, and for typical interior panels (which exist in theory only) are considered independent of the column stiffness. With the Equivalent Frame Analysis, however, the design moments are based on the span equal to the distance between the column centerlines, and are a function of the column cross-sectional properties. The column cross-sectional dimensions influence the design moments in two ways: firstly, by affecting the stiffness factor for the distribution procedure and secondly, by the reduction of the centerline negative moment to that at the column face.

The negative design moments obtained by Direct Design Method and Equivalent Frame Analysis are compared in Fig. 5.2. For the case of $c_1/\ell_1 = 0$, the small discrepancies between the methods resulted from the fact that with the Direct Design Method the distribution of the total static moment into negative and positive regions uses factors 0.65 and 0.35, respectively, instead of the theoretical values 2/3 and 1/3. Also, in this case the Equivalent Frame Analysis gives the values which are almost identical to the elastic solution.

For the case where $c_1/\ell_1 = 0.2$, small discrepancies between the Equivalent Frame Analysis and the elastic solution resulted from the nature of the reduction of the moment to the column face. In this case Corley's[1] expression was used. Positive design moments were not shown since the discrepancies will be of the same order.

5.3.2 Exterior Panel

In the case of exterior panels, in addition to the loading and slab dimensions, consideration is given to the flexural stiffness of the exterior column and the flexural and torsional stiffness of the beams (if any) for both the Direct Design Method and the Equivalent Frame Analysis. All of these variables are readily considered with the Equivalent Frame Analysis, but with the Direct

Design Method no simple expressions which accurately consider all these factors are available. Figs. 5.3, 5.4 and 5.5 illustrate quantitatively the discrepancies in the design moments in the exterior panel obtained by the Direct Design Method and the elastic solution. The design moments in the interior negative, exterior negative and positive regions are discussed separately.

Fig. 5.3 shows that stiffness properties of the beam and the column do not influence interior negative design moments to any great extent. This is not surprising since it is expected that this negative design moment would be similar to that of the typical interior panel. However, it appears that the Direct Design Method underestimates these design moments through all the range of variables considered. Discrepancies are greatest for the flat slab and amount to almost 15% for very stiff columns. Here, one should keep in mind that reinforced concrete is not a linearly elastic material, an assumption on which the elastic solution is based, but is subject to irreversible effects, such as cracking, creep, etc. Therefore, one could consider that a slab for which reinforcement was proportioned according to the elastic solution could approach Code values through the changing of its mechanical properties with the time and loading. Thus these discrepancies could be considered reasonable.

Exterior negative design moment is much more sensitive on the column, beam and slab stiffness. Elastic solutions (Fig. 5.4) show that the aspect ratio, ℓ_2/ℓ_1, and the flexural stiffness, H, of the exterior beam do not influence design moments extensively. In section 13.3.3.3 of ACI 318-71 this moment is presented as a linear function of the nondimensional factor, $1/(1 + 1/Ke')$ and the effect of the beam torsional stiffness, R, is neglected. Elastic analysis shows that consideration of this latter factor is essential as illustrated in Fig. 5.4. This is the prime reason why the Code values are almost incomparable with the elastic solutions for all types of slabs. Inclusion of the torsional beam stiffness was suggested by Gamble[10] by altering the definition of K' by using in the numerator the term Kec rather than the term Kc, where Kec is defined by equation 13.5[5]. However, it would appear that this change would not give solutions which would be closer to the elastic solutions for all values of the column and beam stiffnesses. Essentially the moment cannot be taken as a linear function of the factor $1/(1 + 1/Ke')$ regardless of whether Ke' includes the effect of R. It is proposed here that the effect of Ke' as defined by ACI 318-71 and R be taken separately by using a formula of this form

$$Ke = K_1[1/(1+Ke')]^{1/n_1} + K_2[1/(1+1/R)]^{1/n_2}$$

where Ke = negative design moment coefficient for the exterior panel

K_1, K_2, n_1 and n_2 = constants.

To fit the elastic solutions, selection of the above constants as $K_1 = 0.30$, $K_2 = 0.35$, and $n_1 = n_2 = 4$ would give:

$$Ke = 0.30\sqrt[4]{1/(1+1/Ke')} + 0.35\sqrt[4]{1/(1+1/R)} \qquad (5.1)$$

Solutions obtained by the Direct Design Method, elastic analysis and analysis by formula (5.1) are compared in Table 5.1. It can be seen that formula (5.1) is in very good agreement with the elastic solution through the whole range of the variables involved.

Fig. 5.5 presents the coefficients for the positive design moments. Discrepancies between the Direct Design Method and the elastic solution are almost of the same magnitude, but are the opposite sense to those observed for the negative interior moment (Fig. 5.3). For the elastic solutions the maximum positive moments, rather than midspan moments are taken, because the critical section moves from midspan towards the exterior column as the stiffnesses of the exterior column and beam decrease. This offset is of the order of one-tenth of the span, and for practical reasons, the Code is justified in specifying midspan since the reinforcement for the maximum positive moment will cover this region.

5.4 Distribution of Moments

5.4.1 Slab-Beam Interaction

In the previous sections it was stated that the beam stiffness properties, H and R, and the panel aspect ratio, ℓ_2/ℓ_1, have little or no effect on the total static moment, positive and negative design moments, except for the negative design moment at the exterior column. However, the proportioning of the negative and positive design moments into column and middle strip moments, and further splitting of the column strip moment into beam and slab moments is governed essentially by the flexural and torsional beam stiffnesses and the panel aspect ratio. Figs. 5.6 through 5.9 show the influence of these parameters on the distribution of moments for an interior and exterior panel.

The effect of the beam stiffness and panel aspect ratio on the negative column strip moment for an interior panel is shown in Fig. 5.6. For a slab without beams the percentage of the total negative design moment assigned to the column strip may be represented by two straight lines: one for the values of the panel aspect ratio, ℓ_2/ℓ_1, less than unity, and one for the values greater than unity. The reason for the flat curve for aspect ratios between 1 and 2 is due to the definition of the column strip; namely, one half of the shorter span ℓ_1.

Introducing a beam only in the direction in which moments are to be evaluated will cause an increase in the proportion of the negative moment assigned to the column strip, as shown in Fig. 5.6 a. The effect of increasing beam stiffness H_1 is to increase moment coefficient in a geometrically decreasing rate.

Introducing a beam also in the transverse direction will decrease the moment assigned to the column strip, since there is a tendency to distribute the negative moment more uniformly across the panel. This effect becomes more pronounced as the ratio of the stiffness of the transverse beam to the stiffness of the longitudinal beam increases, as shown in Fig. 5.6. At the limiting case where H_2 is very much larger than H_1 the slab approaches one-way behavior and the moment is distributed almost uniformly across the panel.

The proportion of the positive design moment assigned to the column strip is shown in Fig. 5.7. It is noted that the effect on the slab behavior by the introduction of beams in both the longitudinal and transversal directions is identical to that observed for the negative column strip moment. The prime difference is that for a slab without beams, the positive moment at midspan is more uniformly distributed across the panel. This results in assigning substantially less percentage of the total positive

design moment to the column-strip than was the case for the negative moment. However, this reduction does not exist to any great extent when beams are present.

For the exterior panels, the total moment across the discontinuous edge depends primarily on the magnitude of the stiffnesses of the exterior column and the longitudinal beam, but the distribution of this moment across the edge depends primarily on the magnitudes of the flexural and torsional stiffnesses of the edge beam. The percentage of total negative design moment at the exterior edge assigned to the column strip is shown in Fig. 5.8. In this figure, the stiffness of the exterior column corresponding to a medium stiff column was kept constant ($K_c=3K_s$). The effect of the flexural stiffness of the longitudinal beam, H_1, was seen to be negligible, except for the small values of the span ratio ℓ_2/ℓ_1 where an increase in the H_1 causes a small increase in the percentage of moment assigned to the column strip. Since in practice the flexural and torsional stiffnesses of the edge beam are frequently similar, they are considered equal in Fig. 5.8. Increasing the stiffnesses of the edge beam causes a more uniform distribution of the moments. Varying the stiffnesses of this beam from zero to infinity decreases the moment assigned to the column strip from 100% to uniform distribution, as indicated by the dotted lines in Fig. 5.8.

The percentage of the negative column strip moment assigned to the longitudinal beam increases as the stiffness of this beam increases, as shown in Fig. 5.9. It can be noted that the percentage of the design moment assigned to the beam is essentially independent of the ratio of transverse to longitudinal beam stiffness and the panel aspect ratio.

5.4.2 Slabs with Beams in One Direction Only

This type of slab does not meet the limitations given in section 13.3.1.6 of ACI 318-71, and therefore cannot be analysed by Code procedures. The purpose of this section is to examine the influences of the beam stiffness properties and the aspect ratio on the design moment for this type of slab, with the intention of extending, in later sections, the design procedures to cover this case. This type of slab is essentially a special case of those discussed in section 5.4.1 and for the sake of clarity is presented separately.

From Figs. 5.10 and 5.11 it can be seen that when the beam is very stiff, almost all of the moment, in the direction parallel to the beam, is carried in the column strip containing the beam, whereas in the direction perpendicular to this beam, the moment is distributed almost uniformly, indicating one-way behavior, as expected.

As the stiffness of the beam decreases, the moments in the column strip containing the beam decrease and the moments in the column strip perpendicular to the beam increase. The proportion of the column strip assigned to the beam is given in Fig. 5.12. It is observed that, for very stiff beams, almost all of the moment is carried by the beam in both the negative and positive regions. As the beam stiffness is decreased the proportion carried by the beam decreases but at a faster rate in the positive region. Any procedure which is proposed for the design of slabs with the beam in one direction only must account for the behavior discussed above.

5.4.3 Comparison of Proposed Requirements with Elastic Solutions.

5.4.3.1 General Remarks

In sections 5.4.1 and 5.4.2 the behavior of slabs supported on beams and columns was discussed based on elastic solutions. Parameters representing individual stiffnesses of the supporting beams and columns and panel and column aspect ratios were varied individually to facilitate the interpretation of their effects. In this section design moments obtained by proposed ACI 318-71 are compared with corresponding moments obtained from an elastic analysis. A comparison is made using the dimension-

less stiffness parameters defined by the Code. Comparison is restricted to the column strip moments, it being understood that the remaining part of the total design moment is assigned to the middle strip. Similarly, discussion is restricted to the portion of the column strip moment which is assigned to the beam, the remaining portion going to the slab.

5.4.3.2 Negative Column Strip Moment

The percentage of the interior negative design moment to be assigned to the column strip by both the Direct Design Method and Equivalent Frame Analysis is given in section 13.3.4.1[5]. This percentage is a function of ℓ_2/ℓ_1 and H_1 with no limitations for the Equivalent Frame Analysis, but with the limitations given in section 13.3.1 for the Direct Design Method. The Solid lines in Fig. 5.13 represent values obtained by the Code[5] and the dashes the values obtained by elastic solution. It can be seen that there is very good agreement for the square panel (ℓ_2/ℓ_1 = 1) for all the values of the beam flexural stiffness, H_1. However, for the rectangular panels without beams, the Code overestimates the negative moment; for example, by 12% when ℓ_2/ℓ_1=0.5. This means that the middle strip will be underdesigned. Better correlation would be obtained if the Code assumed two linear regions:

one for $0.5 \leq \ell_2/\ell_1 \leq 1.0$ and another for $1.0 \leq \ell_2/\ell_1 \leq 2.0$, a condition suggested in the final report of ASCE ACI, Committee 421. However, to limit slab deflections it is desirable that the column-strip moments, rather than the middle-strip moments be overestimated.

In the proposed Code, the proportion of the design moment assigned to the middle and column strip is based on solutions in which the ratio of stiffnesses of the beams in two directions is proportional to the ratio of the corresponding spans. For this condition the ratio $H_1\ell_2^2/H_2\ell_2^2$ is equal to unity(7). However, to encompass the range of stiffnesses usually encountered in practice, the Direct Design Method has established the limits of this ratio to be between 0.2 and 5.0. In an attempt to evaluate the validity of these limits, Fig. 5.14 was plotted. It is observed that the percentage assigned to the column strip increases at a geometrically decreasing rate as the value of this ratio increases. It is seen that the agreement between the Code values and the elastic solution is excellent when parameter $H_1\ell_2^2/H_2\ell_1^2$ is equal to unity. However, for the values greater or lesser than unity, the discrepancies increase particularly for the higher values of the ratio ℓ_2/ℓ_1. For the lower limit 0.2 the Code assigns less than 10% more to the column strip which for practical purposes is reasonable. For the

upper limit, 5.0, the Code underestimates the percentage of negative moment assigned to the column strip and for values of $\ell_2/\ell_1=2$ this underestimation is almost 30% of the negative design moment. Since assigning less negative moment the column strip requires more moment redistribution in the slab and hence greater slab deflections, it would appear that a smaller upper limit, particularly for higher values of ℓ_2/ℓ_1, may be desirable. From Fig. 5.14 it is also seen that the percentage of the negative design moment assigned to the column strip is relatively insensitive to large increases of the stiffness ratio beyond the limits established in the Code.

For the exterior panel the portion of the negative design moment assigned to the column strip for the limiting cases given in the Code is shown in Fig. 5.15. As discussed in section 5.4.3.1 this percentage is a function of the torsional stiffness of the edge beam, R. In general, the agreement between Code and elastic values is satisfactory, the Code overestimating the negative column strip moment for low values of the ratio ℓ_2/ℓ_1 and underestimating for high values

5.4.3.3 Positive Column Strip Moments

The percentage of the positive design moment assigned to the column strip for the interior panel is

given in Fig. 5.16. The discrepancies between the Code and elastic values are similar to those for the negative region and in general are satisfactory for design purposes.

5.4.3.4 Beam Design Moment

The proportion of the column strip moment assigned to the beam is presented in Fig. 5.17. For the range of variables studied there is little difference between the percentage assigned to the beam for the positive and negative regions. Solutions plotted are all for values of the ratio $H_1\ell_2^2/H_2\ell_1^2 = 1.0$. It should be noted that the percentage assigned to the beam is dependent primarily on the ratio $H_1\ell_2/\ell_1$ and increases as this ratio increases. In every case this percentage increases as the ratio ℓ_2/ℓ_1 decreases from unity, but is essentially constant for values greater than unity. Section 13.3.4.4 of ACI 318-71 states that: "The beam shall be proportioned to resist 85 percent of the column strip moment if $H_1\ell_2/\ell_1$ is equal or greater than 1.0. For values of $H_1\ell_2/\ell_1$ between 1.0 and zero the proportion of the moment to be resisted by the beam shall be obtained by linear interpolation between 85 and zero percent." However, Fig. 5.17 shows that there is generally poor agreement between elastic

solutions and Code proposals. For the $H_1\ell_2/\ell_1 = 1.0$ and $\ell_2/\ell_1 = 1.0$ the elastic beam design moments is 62% of the column strip moment or 23% less than Code value. Moreover, the moment does not increase linearly with $H_1\ell_2/\ell_1$ and the Code underestimates the beam moment by 10% when $\ell_2/\ell_1 = 1.0$. This underestimation becomes greater as ℓ_2/ℓ_1 decreases. In an attempt to obtain percentages for the values of $H_1\ell_2/\ell_1$ equal 1.0 and 0.5 and to maintain a simple linear interpolation, it appears that the Code has established the percentage assigned to the beam unreasonably high for $H_1\ell_2/\ell_1 = 1.0$ and low for $H_1\ell_2/\ell_1 = 0.5$. Since cracking and creep will cause redistribution of moment from the beam to the slab, the actual moment in the beam will be even less than that indicated by the Code procedure. This will increase the discrepancy whenever the Code underestimates the beam design moment.

5.5 Suggested Procedure

This procedure follows the philosophy of the Direct Design Method except that it modifies the distribution of moments at the exterior negative region and between column and middle strips to give results that agree more closely with elastic solutions.

In sections 5.2 and 5.3 it was observed that the values of the Code for the total static design moment

for both the interior and exterior panel, and the splitting of this moment into negative and positive regions for an interior panel are in good agreement with those of elastic solution. However, large discrepancies exist in an exterior panel, especially in the case of the exterior negative moment. In section 5.3.2. formula (5.1) was suggested and comparisons between three procedures were made in Table 5.1. It should be noted again that the suggested formula (5.1) was derived to fit the elastic solution. On the other hand, the Code itself is based on elastic slab analyses, some of which were verified by tests. Therefore, the philosophy behind all three procedures compared in Table 5.1 is the same. The fact that ACI 318-71 values are in disagreement with the elastic solution is mainly due to the lack of consideration given to the torsional stiffness of the edge beam, R. It can be seen from Table 5.1 that formula (5.1) has taken this effect into account properly, and hence very good agreement with the elastic solution is obtained.

Better, although not satisfactory, agreement was obtained between the elastic solution and the Code values insofar as the splitting of the design moments in the critical sections is concerned. It would appear that the tables given in section 13.3.4 of ACI 318-71 are oversimplified, especially for the beam design moments. With the general computer program, on which this thesis

is based it would be possible to obtain any desired accuracy by extending these tables even for the slabs which do not meet the Code limitations. However, these extended tables would be significantly larger since they would be based on four variables: H_1, H_2, R, and ℓ_2/ℓ_1, rather than three parameters used by the Code, $H_1\ell_2/\ell_1$, $H_1\ell_2^2/H_2\ell_1^2$, and R. Based on the idea that it is more convenient to distribute design moments in the critical sections using coefficients derived from formulas rather than having to interpolate between table values, the following four formulas are suggested.

$$k_{ni} = 0.70 + 0.05\left(\frac{\ell_2}{\ell_1}\right) + \frac{0.2}{1+\frac{1}{H_1}} - \frac{0.1}{1+\frac{1}{H_2}} - \frac{0.1}{1+\frac{H_1}{H_2}}\left(\frac{\ell_2}{\ell_1}\right)^2 \tag{5.2}$$

$$k_{ne} = 0.80 + \frac{0.2}{1+R} - \frac{0.3}{1+\frac{10}{R+H_2}}\left(\frac{\ell_2}{\ell_1}\right) - \frac{0.1}{1+\frac{10}{H_1+H_2}} \tag{5.3}$$

$$k_{pi} = 0.55 + 0.05\left(\frac{\ell_2}{\ell_1}\right) + \frac{0.35}{1+\frac{1}{2H_1}} - \frac{0.05}{1+\frac{1}{H_2}}\left(\frac{\ell_2}{\ell_1}\right) - \frac{0.05}{1+\frac{H_1}{H_2}}\left(\frac{\ell_2}{\ell_1}\right)^2 \tag{5.4}$$

$$k_b = 0.40 + 0.10\left(\frac{\ell_2}{\ell_1}\right) + \frac{0.4}{1+\frac{1}{2H_1}} - \frac{0.1}{1+\frac{1}{H_2}}\left(\frac{\ell_2}{\ell_1}\right) \tag{5.5}$$

where; k_{ni}, k_{ne}, k_{pi}, k_b are, respectively, coefficients

for the interior negative column strip moment, exterior negative column strip moment, positive column strip moment, and the beam moment. If desired, the coefficients in the above formulas may be converted to percentages by multiplying by 100.

In summary, it should be noted that formula (5.1) could replace the last equation in section 13.3.3.3 of ACI 318-71, and that formulas (5.2) through (5.5) represent alternate solutions for the tables given in section 13.3.4.1, 13.3.4.2, 13.3.4.3 and 13.3.4.4 of the proposed Code[5]. It is important to note that, although these formulas require only the same variables as the Code tables, they are more general in that they are not restricted by the limitations required in section 13.3.1.6 of ACI 318-71. Tables 5.2 through 5.5 compare results obtained by these formulas with ACI 318-71 values and the results of the elastic solution. It can be seen that the results obtained by formulas 5.2 to 5.4 agree much more closely with those of the elastic solution, and that indeed in most cases they slightly overestimate the elastic values. This is an important factor because the moment in question here is in the column strip (including beam) where it could be expected that cracking would take place first. These formulas are not comples and can be simplified, although at the expense of accuracy.

TABLE 5.1

COEFFICIENTS FOR THE EXTERIOR NEGATIVE DESIGN MOMENT OBTAINED BY DIFFERENT PROCEDURES.

$1/(1+1/K_e')$	R	ℓ_2/ℓ_1	ACI 318-71	Elastic Analysis	Suggested Procedure (Eqn. 5.1)
0	0	any	0	0	0
0.25	0	0.5	0.16	0.21	0.21
0.50	0	0.5	0.32	0.23	0.25
0.75	0	0.5	0.49	0.24	0.28
1.00	0	0.5	0.65	0.25	0.30
0.25	0	1.0	0.16	0.22	0.21
0.50	0	1.0	0.32	0.24	0.25
0.75	0	1.0	0.49	0.25	0.28
1.00	0	1.0	0.65	0.26	0.30
0.25	0	2.0	0.16	0.24	0.21
0.50	0	2.0	0.32	0.30	0.25
0.75	0	2.0	0.49	0.31	0.28
1.00	0	2.0	0.65	0.32	0.30
0.25	0.5	1.0	0.16	0.40	0.47
0.50	0.5	1.0	0.32	0.48	0.51
0.75	0.5	1.0	0.49	0.51	0.54
1.00	0.5	1.0	0.65	0.52	0.56
0.25	1.0	1.0	0.16	0.41	0.50
0.50	1.0	1.0	0.32	0.52	0.52
0.75	1.0	1.0	0.49	0.57	0.57
1.00	1.0	1.0	0.65	0.60	0.59
0.25	5.0	1.0	0.16	0.42	0.54
0.50	5.0	1.0	0.32	0.54	0.58
0.75	5.0	1.0	0.49	0.61	0.61
1.00	5.0	1.0	0.65	0.65	0.63
1.00	∞	any	0.65	0.65	0.65

TABLE 5.2

COEFFICIENTS FOR THE NEGATIVE COLUMN-STRIP MOMENT
IN AN INTERIOR PANEL OBTAINED BY DIFFERENT PROCEDURES

H_1	H_2	ℓ_2/ℓ_1	ACI 318-71	Elastic Analysis	Suggested Procedure (Eqn. 5.2)
0	0	0.5	0.75	0.63	0.72
0	0	1.0	0.75	0.76	0.75
0	0	1.5	0.75	0.76	0.77
0	0	2.0	0.75	0.76	0.80
1.0	0.5	0.5	0.82	0.77	0.78
1.0	1.0	1.0	0.75	0.75	0.75
5.0	2.5	0.5	0.90	0.81	0.87
5.0	5.0	1.0	0.75	0.74	0.75
1.0	5.0	2.0	0.45	0.43	0.48
0.5	2.5	2.0	0.45	0.47	0.46
0	∞	2.0	–	0.25	0.30
0.5	0	0.5	–	0.77	0.79
1.0	0	0.5	–	0.83	0.83
5	0	0.5	–	0.94	0.89
0.5	0	1.0	–	0.87	0.82
1.0	0	1.0	–	0.91	0.85
5.0	0	1.0	–	0.97	0.92
0	0.5	1	–	0.65	0.67
0	1.0	1	–	0.67	0.60
0	10.0	1	–	0.52	0.56
0	∞	1	–	0.50	0.55

TABLE 5.3

COEFFICIENTS FOR EXTERIOR NEGATIVE COLUMN-STRIP MOMENT IN THE EXTERIOR PANEL OBTAINED BY DIFFERENT PROCEDURES.

H_1	H_2	R	ℓ_2/ℓ_1	ACI 318-71	Elastic Analysis	Suggested Procedure (Eqn. 5.3)
0.5	0	0.5	0.5	–	0.85	0.92
0.5	0	0.5	1.0	–	0.96	0.91
0.5	0	0.5	1.5	–	1.00	0.91
0.5	0	0.5	2.0	–	1.00	0.91
0.5	2	2.5	2.0	0.55	0.61	0.65
0.67	1.5	2.5	1.5	0.64	0.68	0.71
1.0	1.0	2.5	1.0	0.73	0.78	0.76
2.0	0.5	2.5	0.5	0.82	0.85	0.80
0	0	2.5	2.0	0.75	0.85	0.70
0	0	2.5	1.5	0.75	0.83	0.73
0	0	2.5	1.0	0.75	0.81	0.78
0	0	0	any	1.00	1.00	1.00
1.0	1.0	1.0	0.5	0.82	0.80	0.85
1.0	1.0	1.0	1.0	0.73	0.83	0.83
1.0	1.0	1.0	1.5	0.64	0.80	0.80
1.0	1.0	1.0	2.0	0.55	0.79	0.78
5.0	5.0	5.0	0.5	0.82	0.87	0.70
5.0	5.0	5.0	1.0	0.73	0.70	0.64
5.0	5.0	5.0	1.5	0.64	0.45	0.55
5.0	5.0	5.0	2.0	0.55	0.33	0.48

TABLE 5.4

COEFFICIENTS FOR THE POSITIVE COLUMN STRIP MOMENT OBTAINED BY DIFFERENT PROCEDURES.

H_1	H_2	ℓ_2/ℓ_1	ACI 318-71	Elastic Analysis	Suggested Procedure (Eqn. 5.4)
0.0	0.0	0.5	0.60	0.50	0.57
0.0	0.0	1.0	0.60	0.60	0.62
0.0	0.0	1.5	0.60	0.59	0.67
0.0	0.0	2.0	0.60	0.59	0.70
0.5	0.0	0.5	–	0.68	0.72
0.5	0.0	1.0	–	0.78	0.74
0.5	0.0	1.5	–	0.80	0.77
0.5	0.0	2.0	–	0.85	0.80
1.0	0.0	0.5	–	0.76	0.77
1.0	0.0	1.0	–	0.85	0.80
1.0	0.0	1.5	–	0.87	0.83
1.0	0.0	2.0	–	0.90	0.84
5.0	0.0	0.5	–	0.92	0.86
5.0	0.0	1.0	–	0.94	0.90
5.0	0.0	1.5	–	0.95	0.92
5.0	0.0	2.0	–	0.97	0.93
0.5	0.5	0.5	0.67	0.67	0.73
0.5	0.5	1.0	0.75	0.71	0.75
0.5	0.5	1.5	0.82	0.67	0.77
0.5	0.5	2.0	0.90	0.66	0.76
1.0	0.5	0.5	0.75	0.75	0.77
1.0	0.5	1.0	0.67	0.77	0.82
1.0	0.5	1.5	0.60	0.74	0.84
1.0	0.5	2.0	0.45	0.73	0.84
1.0	1.0	0.5	0.90	0.75	0.79
1.0	1.0	1.0	0.75	0.74	0.80
1.0	1.0	1.5	0.60	0.67	0.81
1.0	1.0	2.0	0.45	0.63	0.78

TABLE 5.5

COEFFICIENTS FOR THE BEAM DESIGN MOMENTS
OBTAINED BY DIFFERENT PROCEDURES.

H_1	H_2	$\ell_2\ \ell_1$	ACI 318-71	Elastic Analysis	Suggested Procedure (Eqn. 5.5)
0.5	0.0	0.5	-	0.53	0.58
0.5	0.0	1.0	-	0.62	0.63
0.5	0.0	1.5	-	0.70	0.68
0.5	0.0	2.0	-	0.77	0.73
1.0	0.0	0.5	-	0.72	0.72
1.0	0.0	1.0	-	0.77	0.77
1.0	0.0	1.5	-	0.82	0.82
1.0	0.0	2.0	-	0.87	0.87
5.0	0.0	0.5	-	0.92	0.82
5.0	0.0	1.0	-	0.94	0.87
5.0	0.0	1.5	-	0.96	0.92
5.0	0.0	2.0	-	0.98	0.97
1.0	0.5	0.5	0.42	0.68	0.70
1.0	0.5	1.0	0.85	0.70	0.73
1.0	0.5	1.5	0.85	0.76	0.77
1.0	0.5	2.0	0.85	0.82	0.80
1.0	1.0	0.5	0.42	0.68	0.69
1.0	1.0	1.0	0.85	0.70	0.72
1.0	1.0	1.5	0.85	0.75	0.75
1.0	1.0	2.0	0.85	0.80	0.78
1.0	5.0	0.5	0.42	0.67	0.69
1.0	5.0	1.0	0.85	0.65	0.69
1.0	5.0	1.5	0.85	0.69	0.70
1.0	5.0	2.0	0.85	0.72	0.71

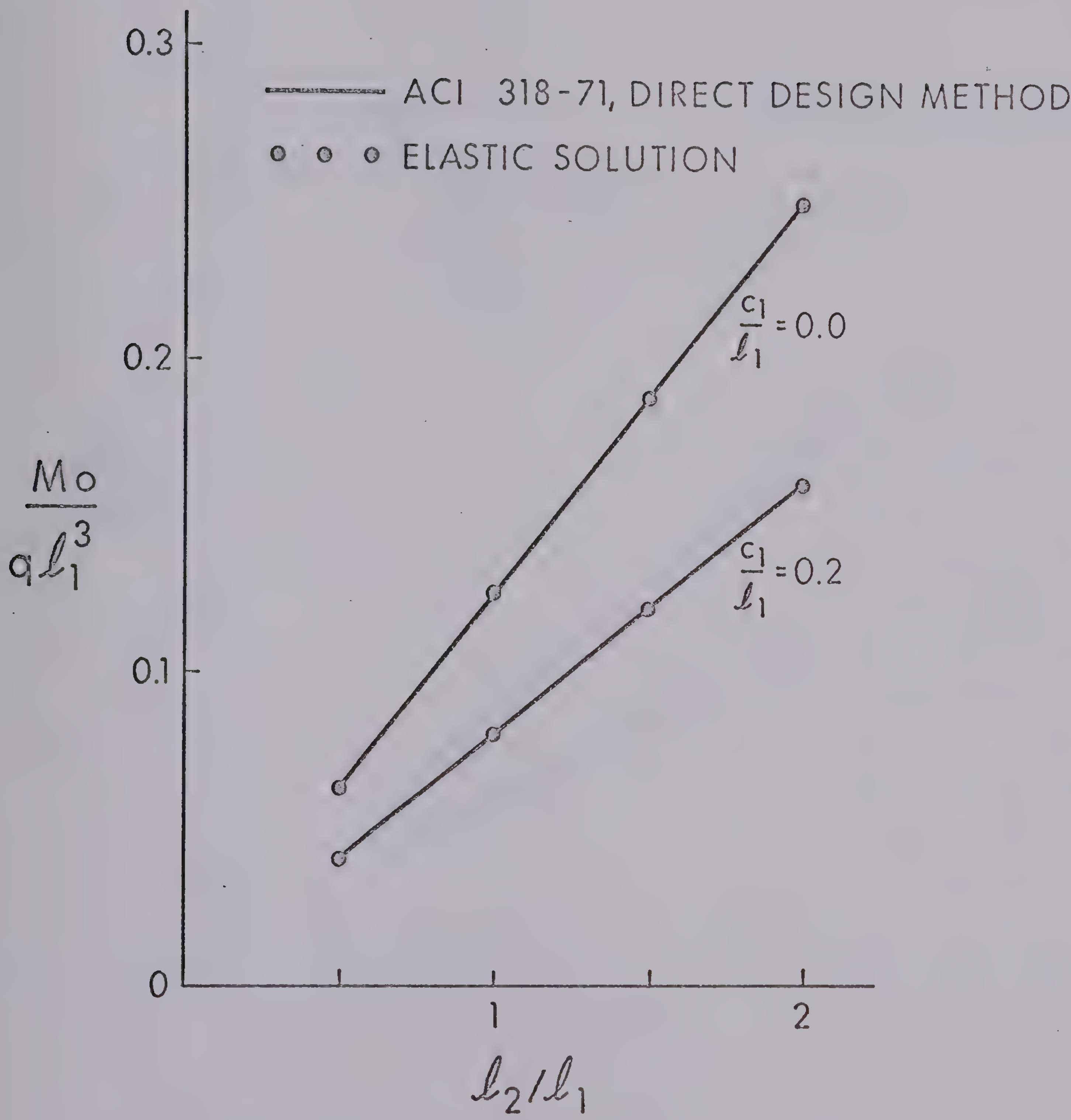


FIGURE 5.1 TOTAL STATIC DESIGN MOMENT

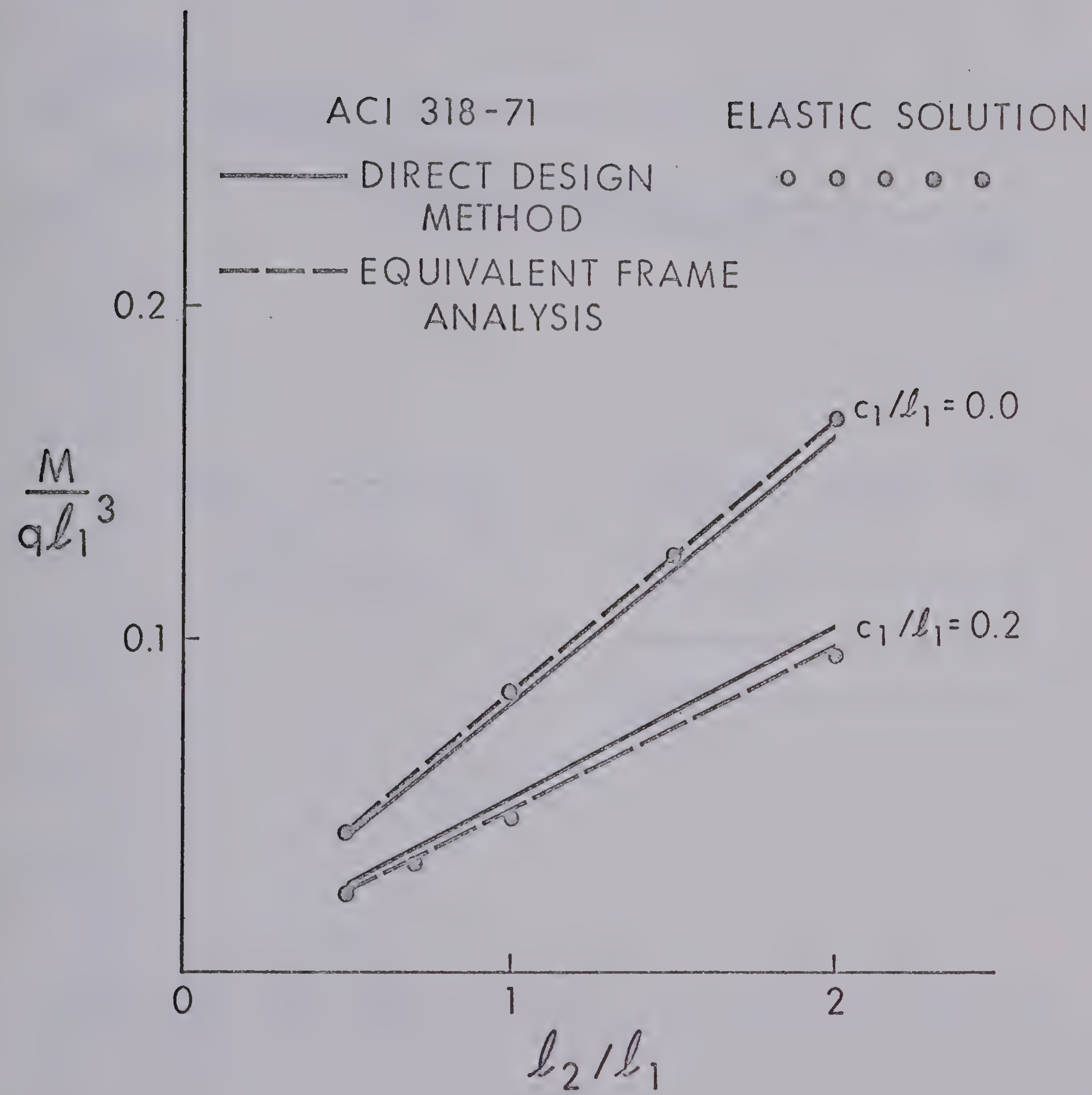


FIGURE 5.2 NEGATIVE DESIGN MOMENT, INTERIOR PANEL

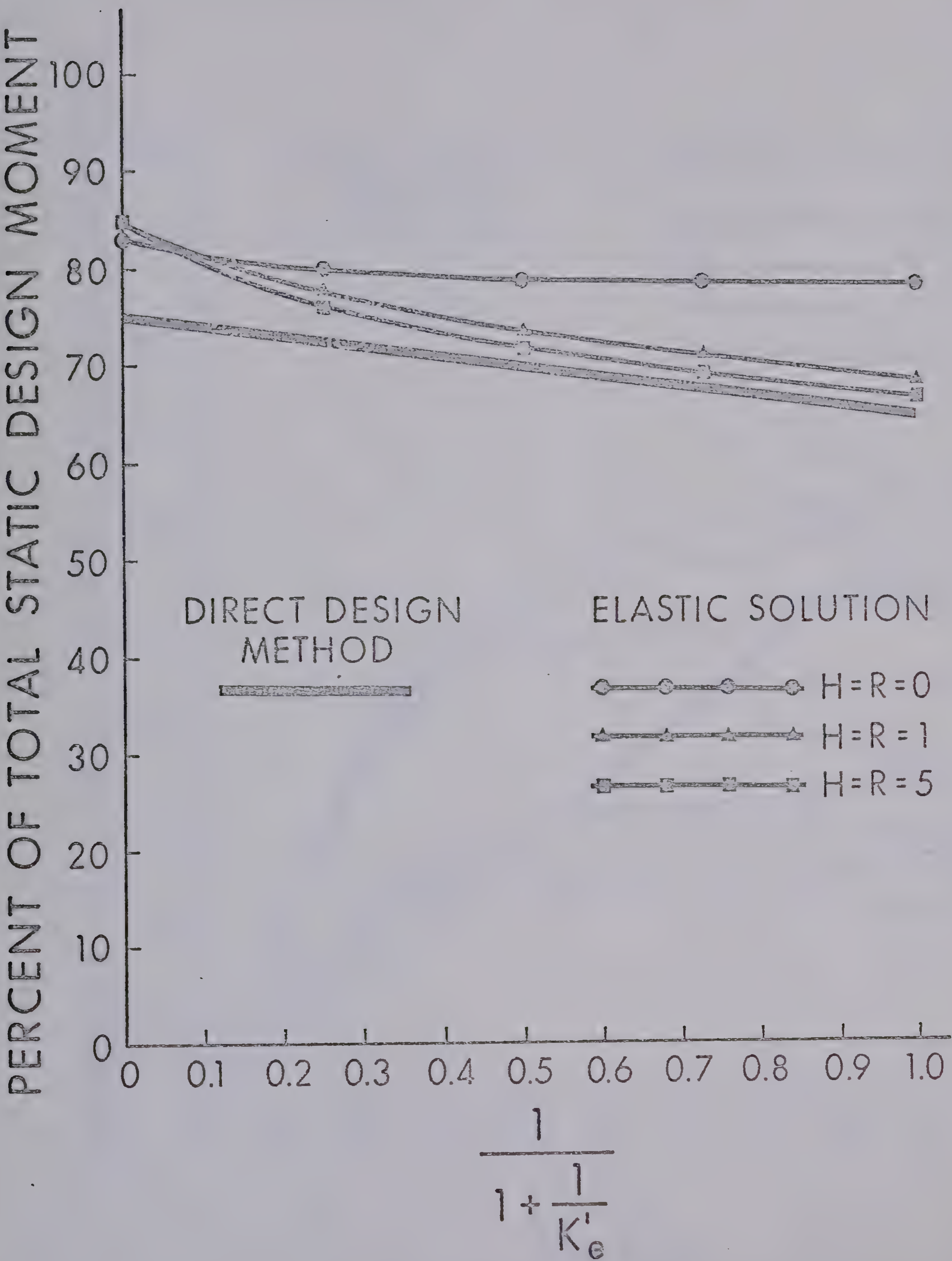


FIGURE 5.3 INTERIOR NEGATIVE DESIGN MOMENT, EXTERIOR PANEL

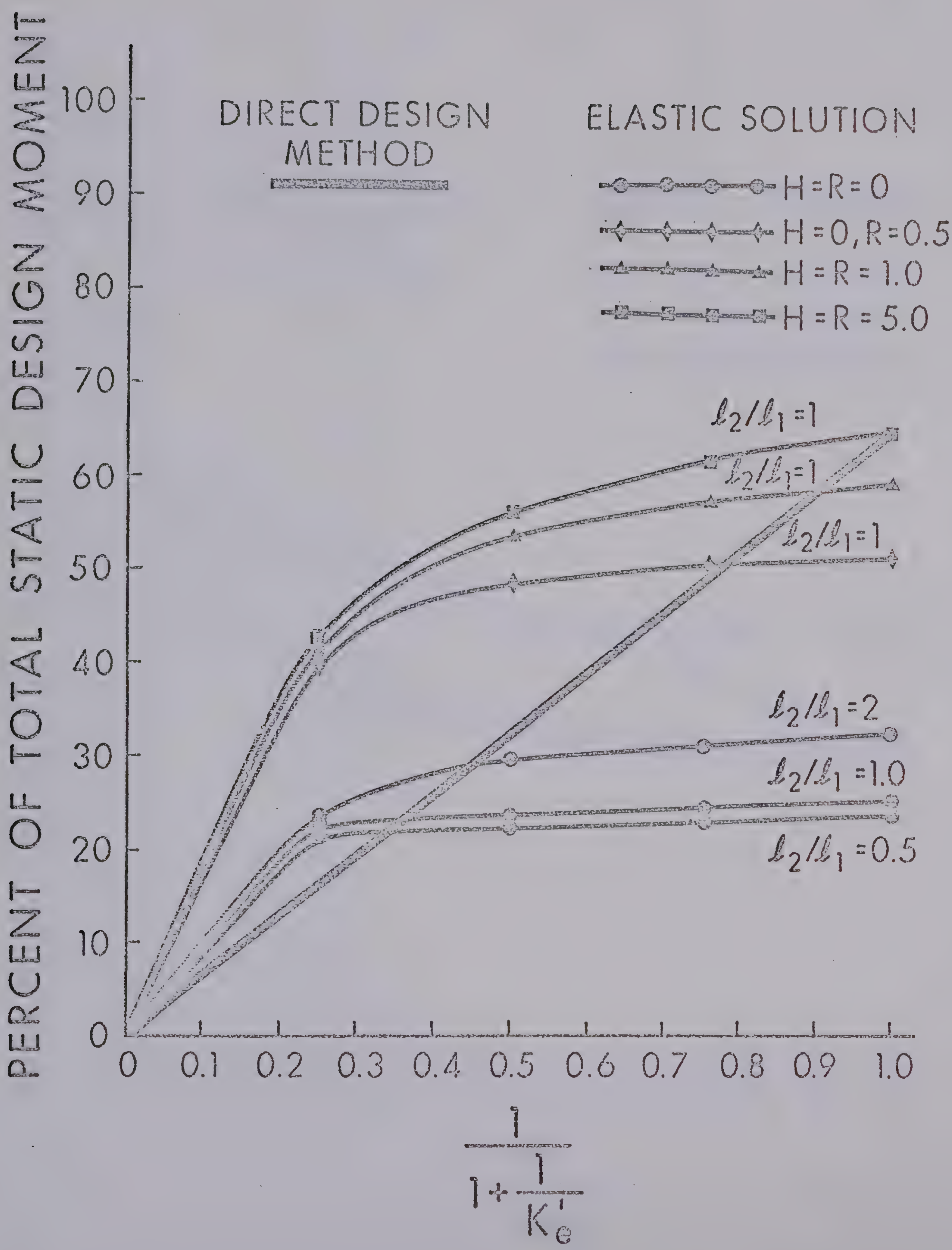


FIGURE 5.4 EXTERIOR NEGATIVE DESIGN MOMENT, EXTERIOR PANEL

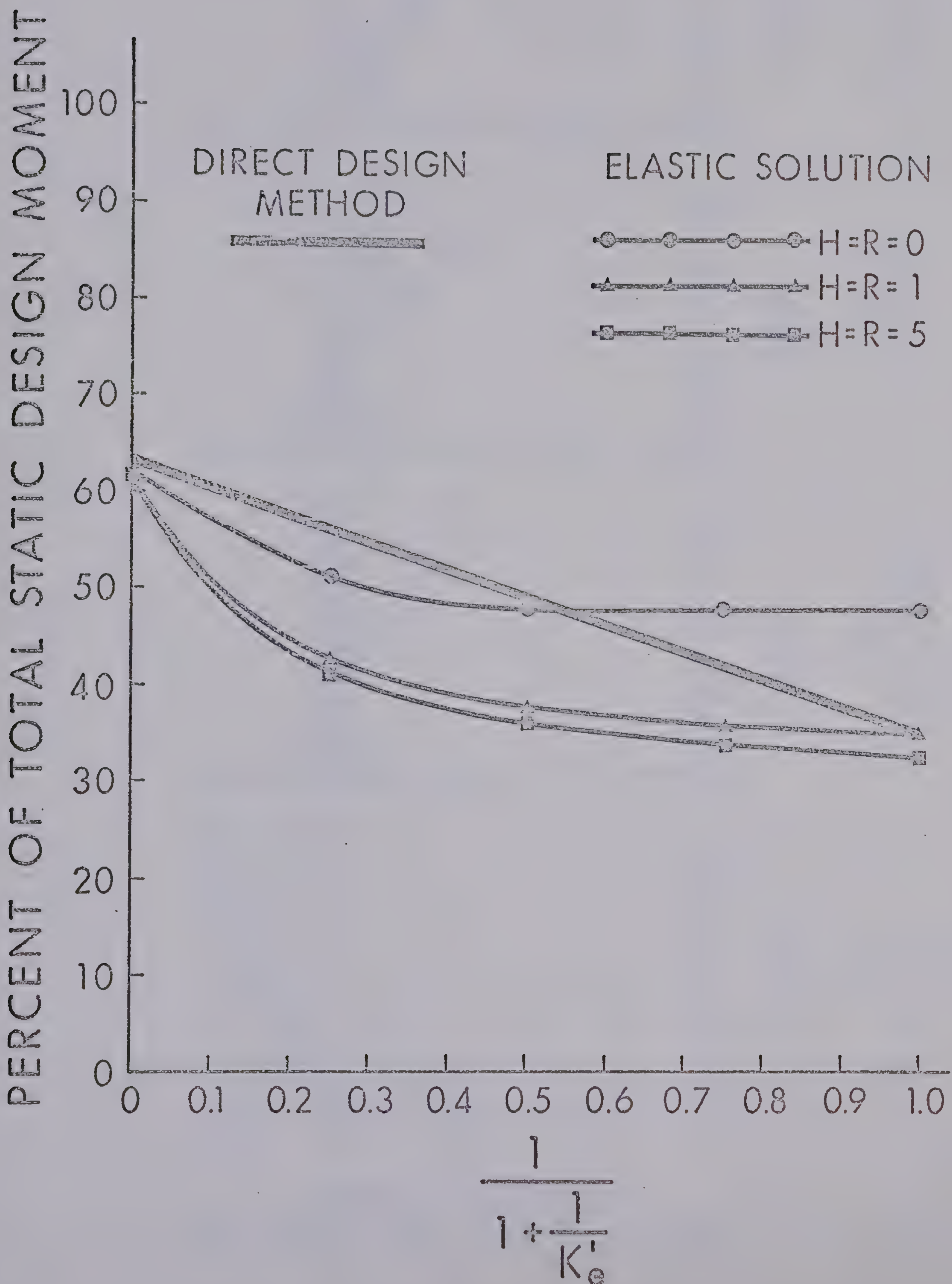


FIGURE 5.5 POSITIVE DESIGN MOMENT, EXTERIOR PANEL

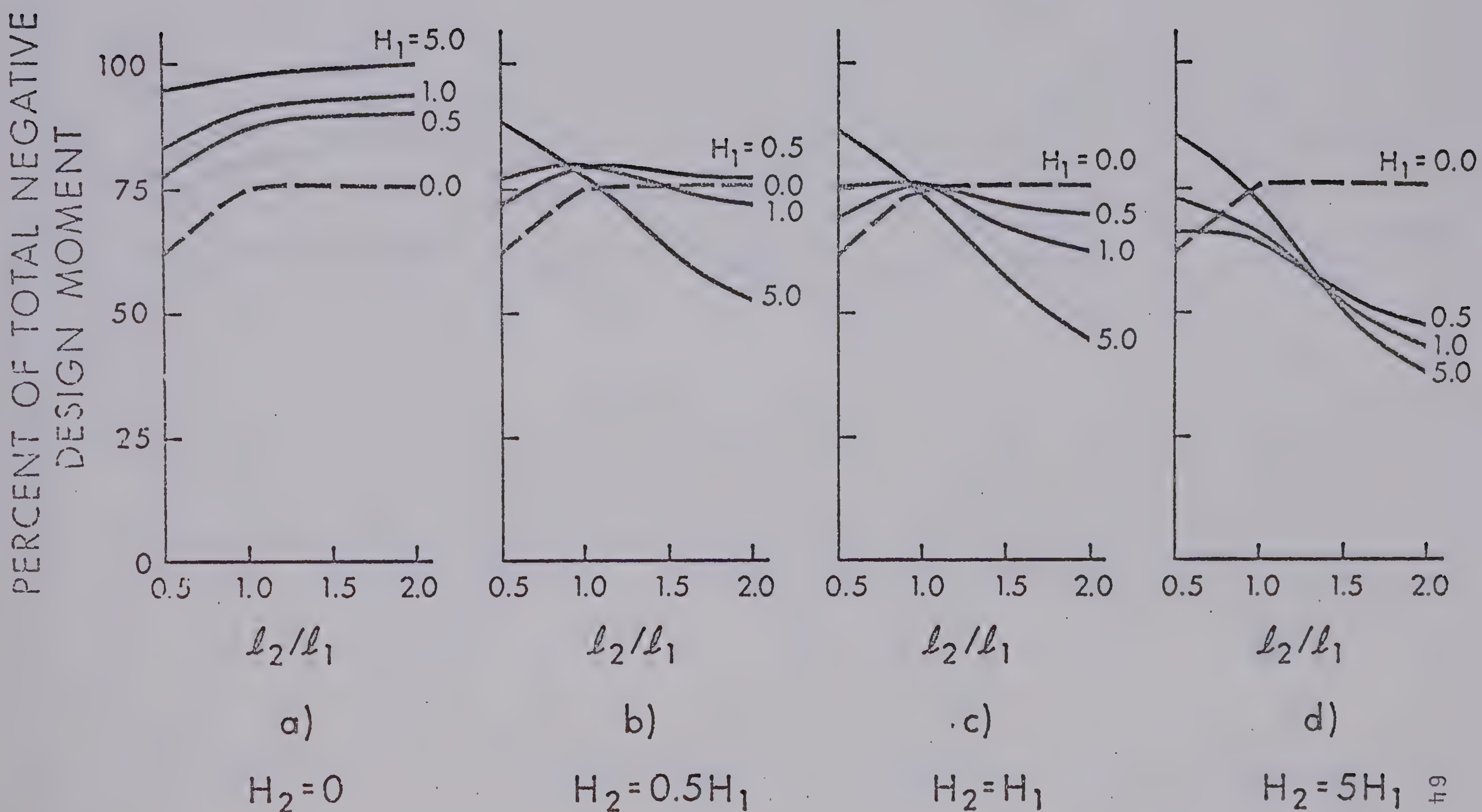


FIGURE 5.6 EFFECT OF VARIABLES ON NEGATIVE COLUMN-STRIP MOMENT IN AN INTERIOR PANEL

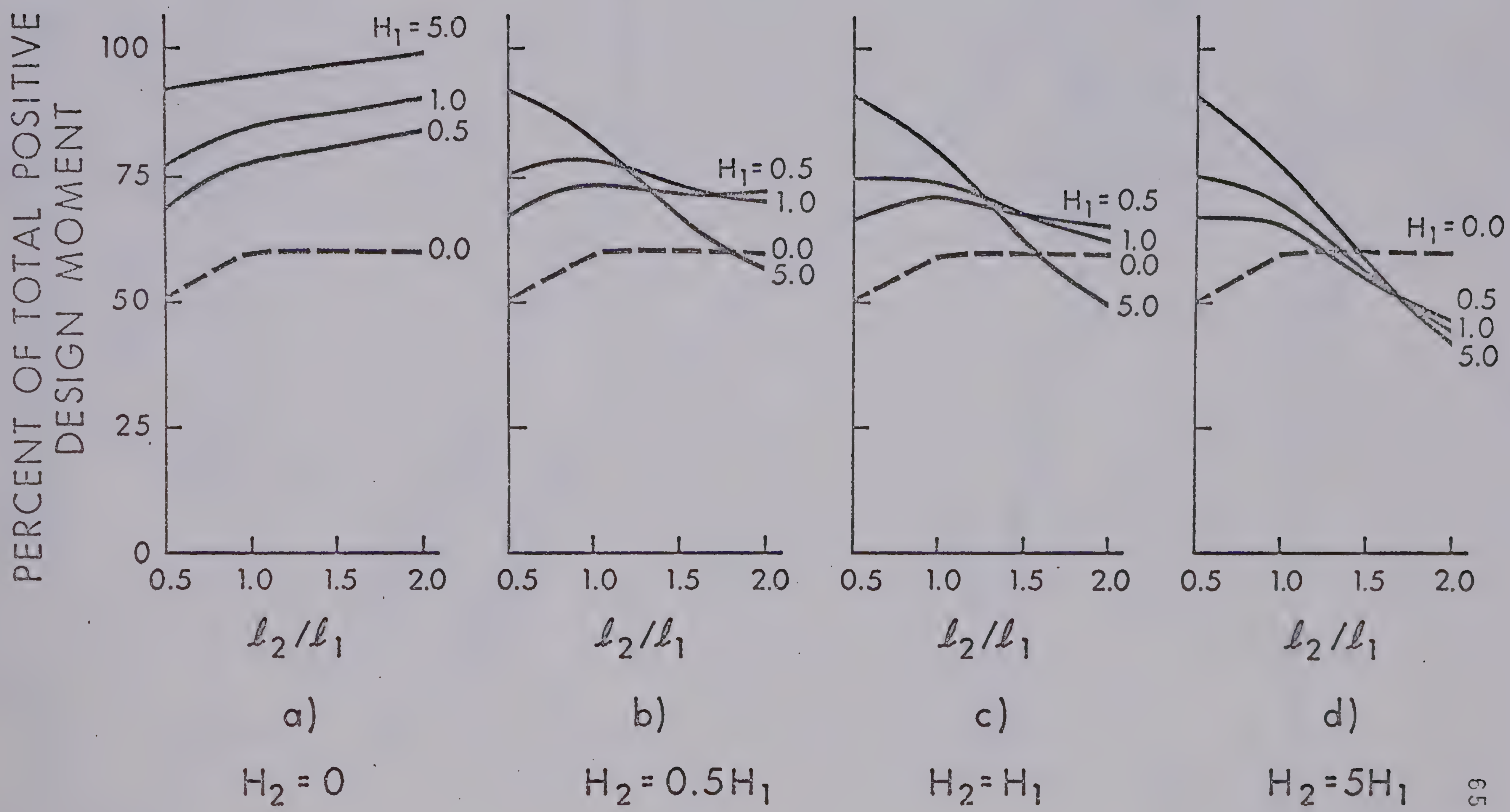


FIGURE 5.7 EFFECT OF VARIABLES ON POSITIVE COLUMN-STRIP MOMENT

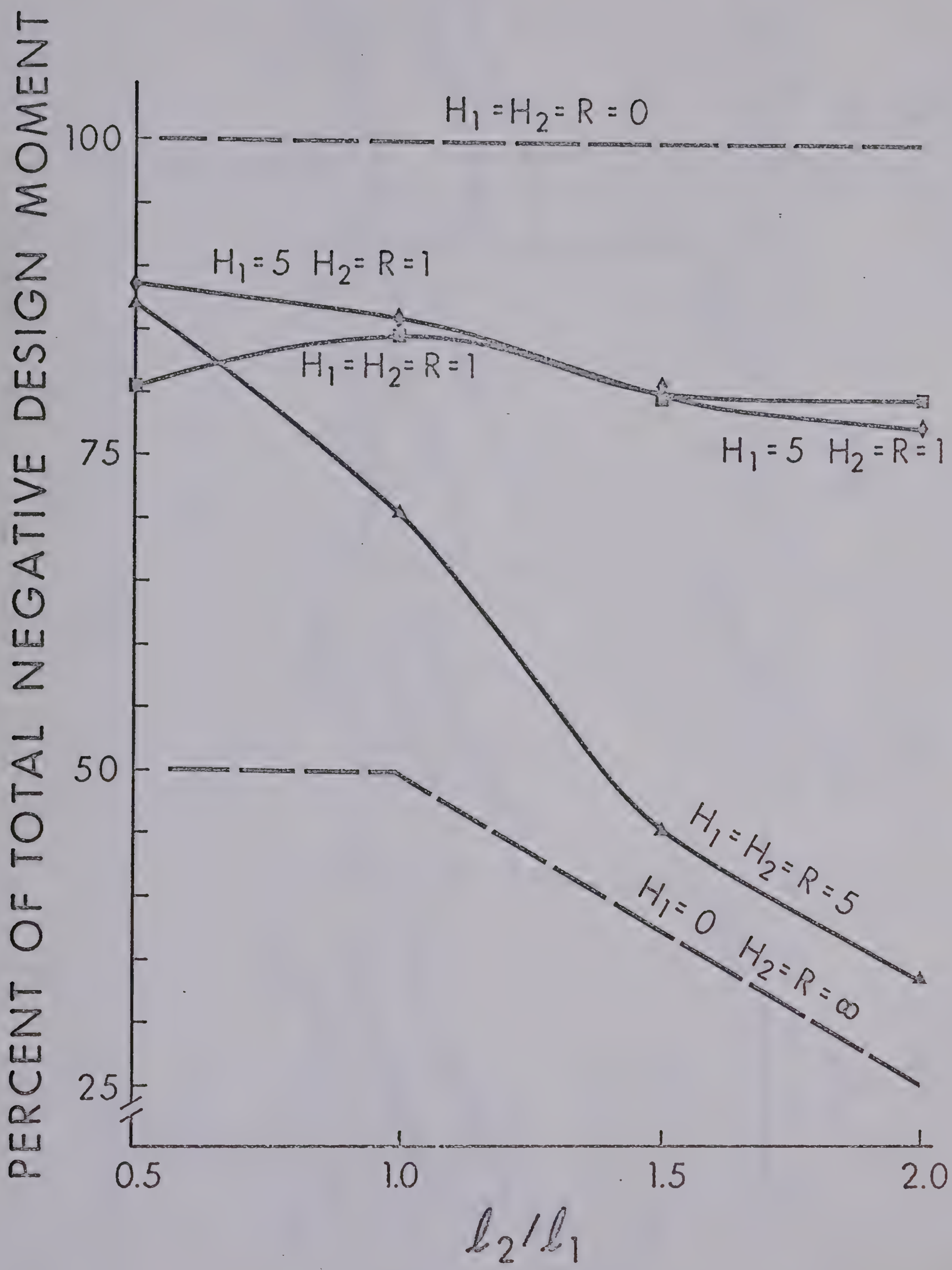


FIGURE 5.8 EFFECT OF VARIABLES ON NEGATIVE COLUMN-STRIP MOMENTS IN AN INTERIOR PANEL, $K_c = 3K_s$

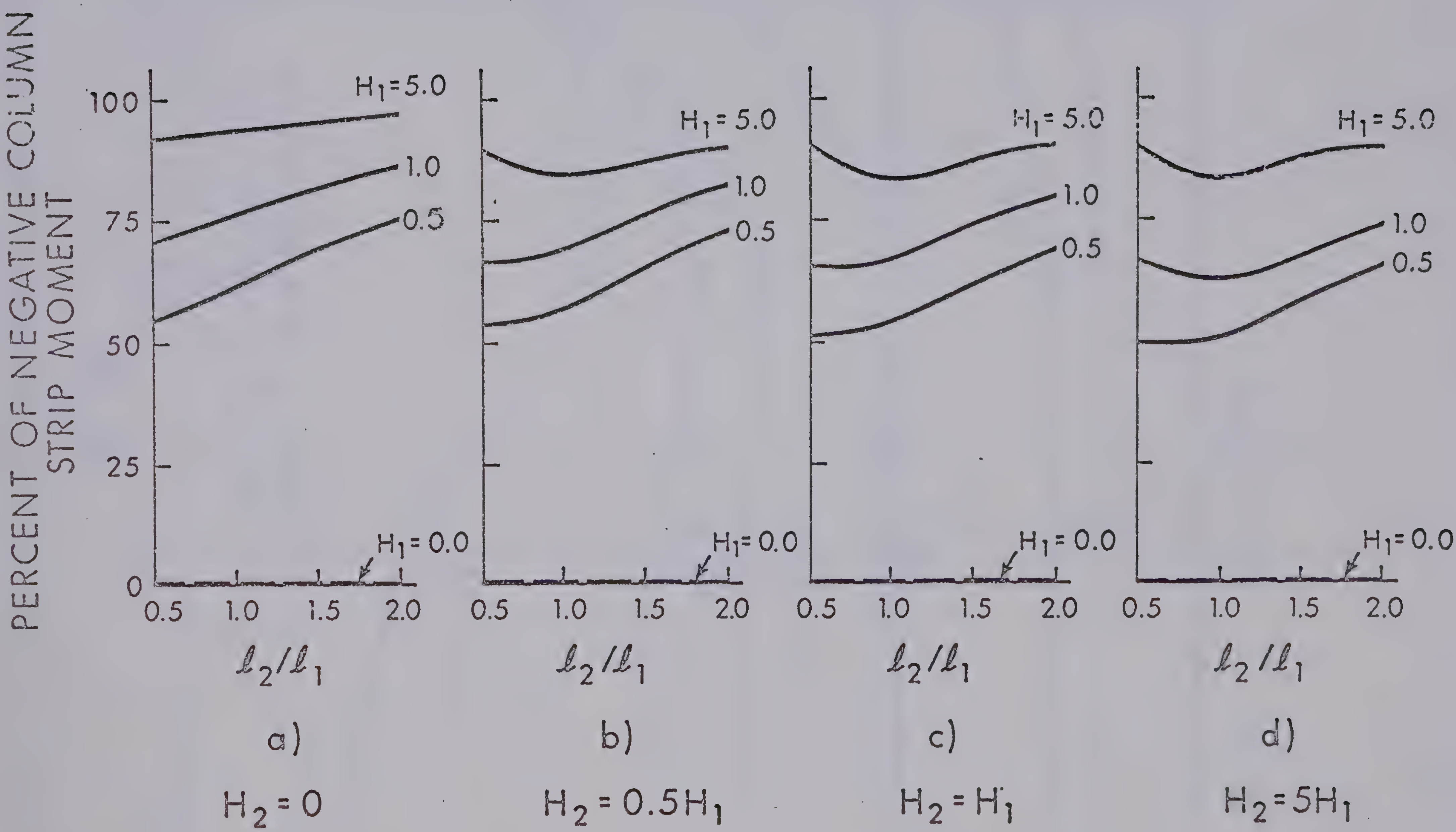


FIGURE 5.9 EFFECT OF VARIABLES ON BEAM DESIGN MOMENT

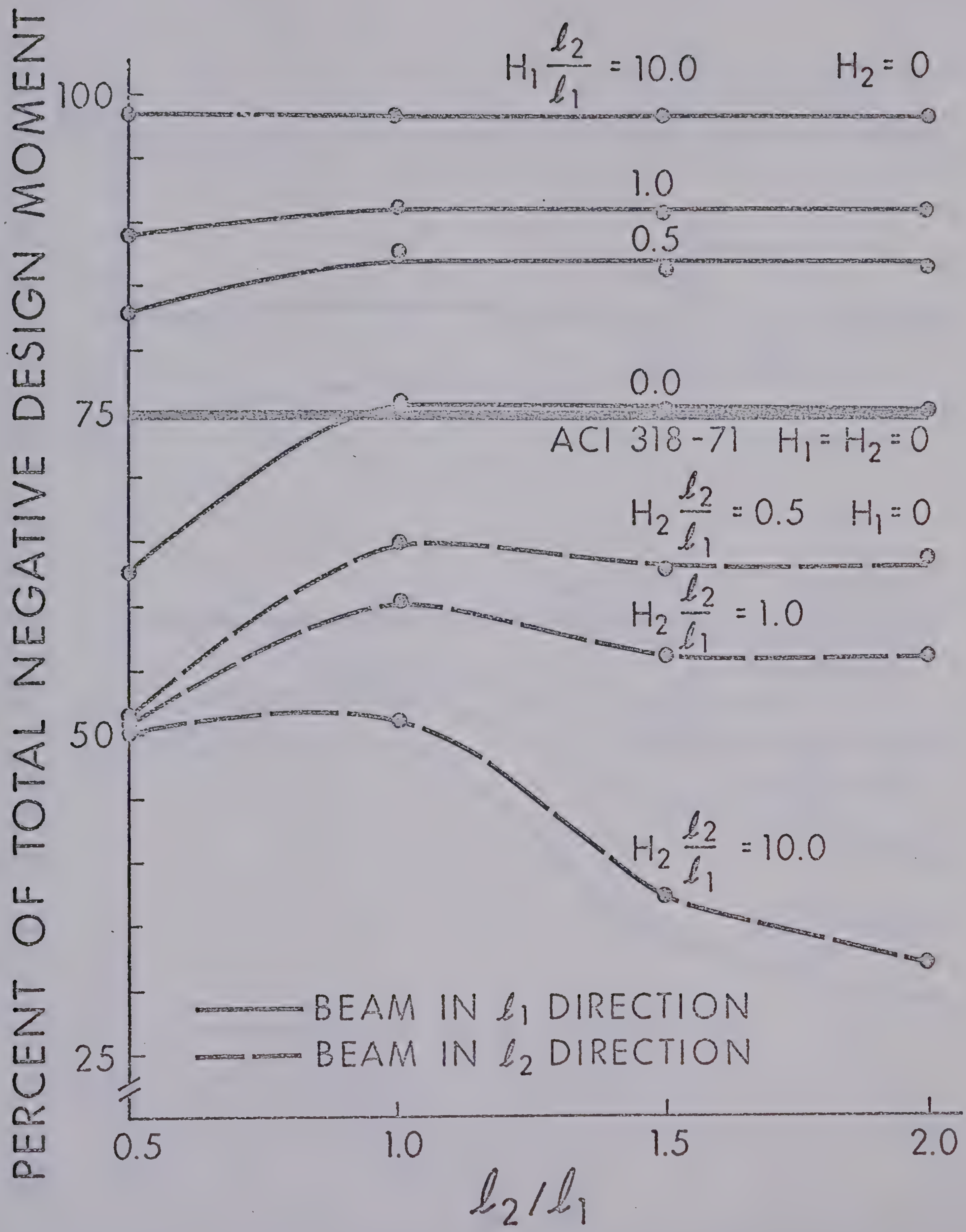


FIGURE 5.10 NEGATIVE COLUMN-STRIP MOMENT, INTERIOR PANEL, BEAM IN ONE DIRECTION ONLY

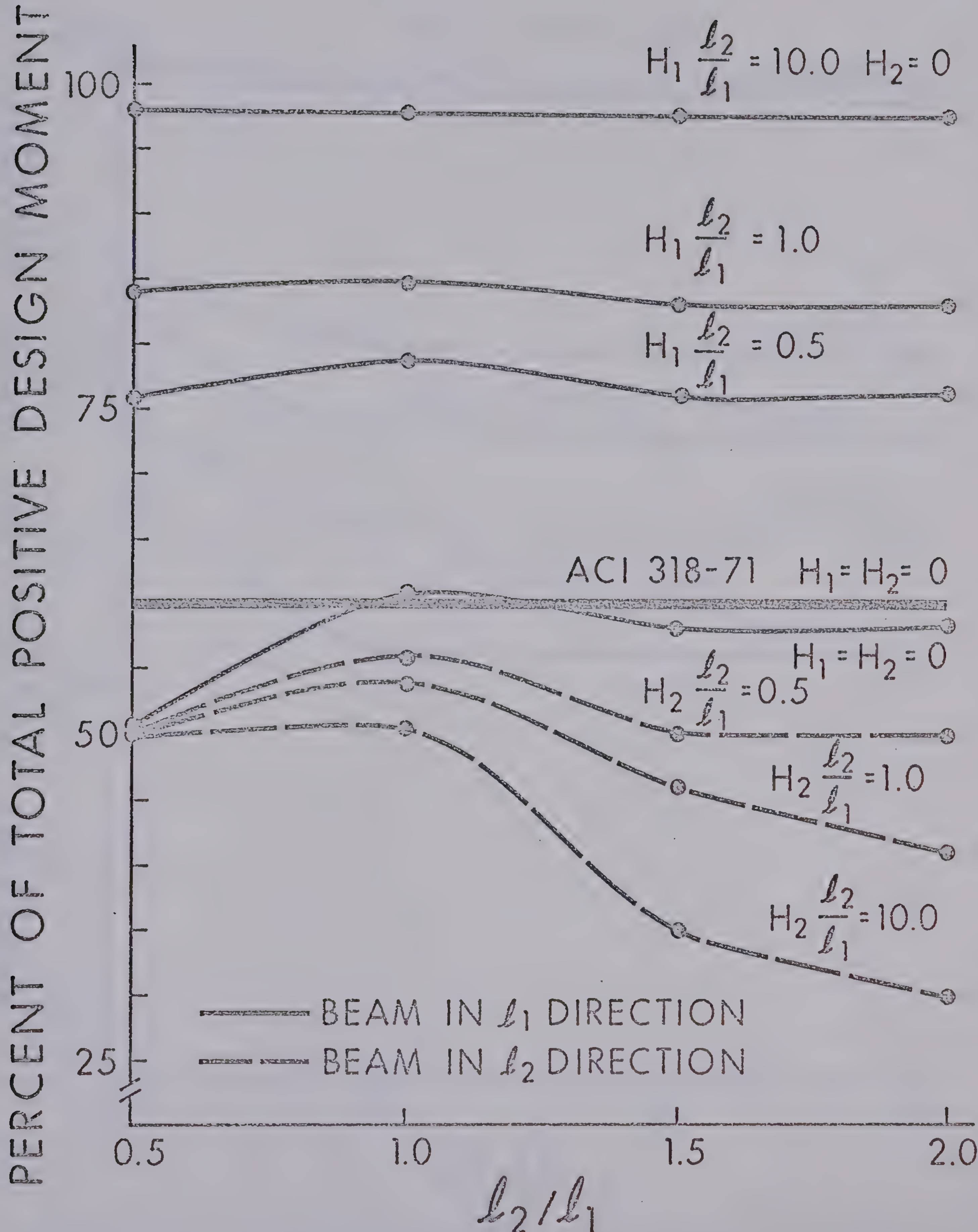


FIGURE 5.11 POSITIVE COLUMN-STRIP MOMENT,
BEAM IN ONE DIRECTION ONLY

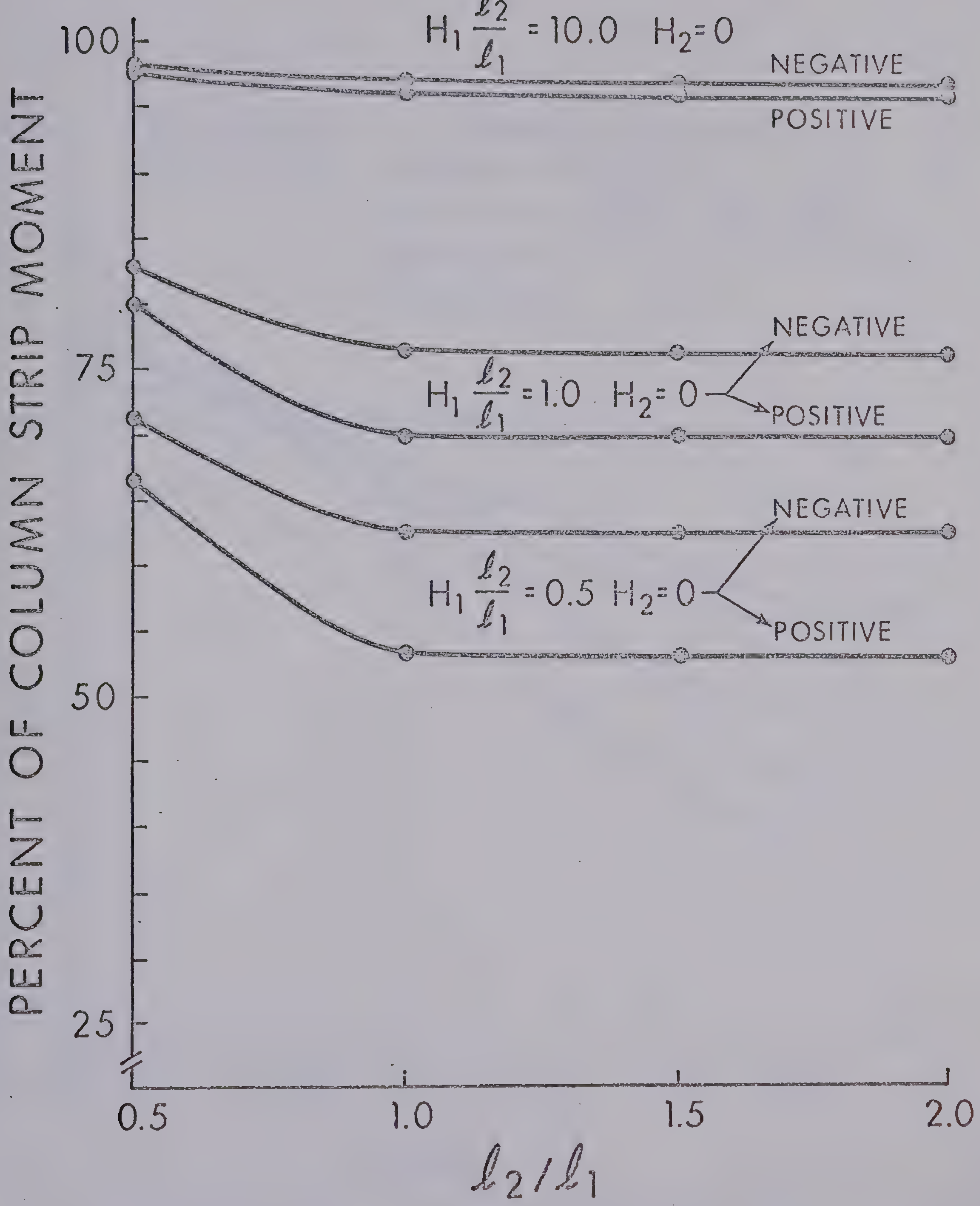


FIGURE 5.12 BEAM DESIGN MOMENT, BEAM IN ONE DIRECTION ONLY

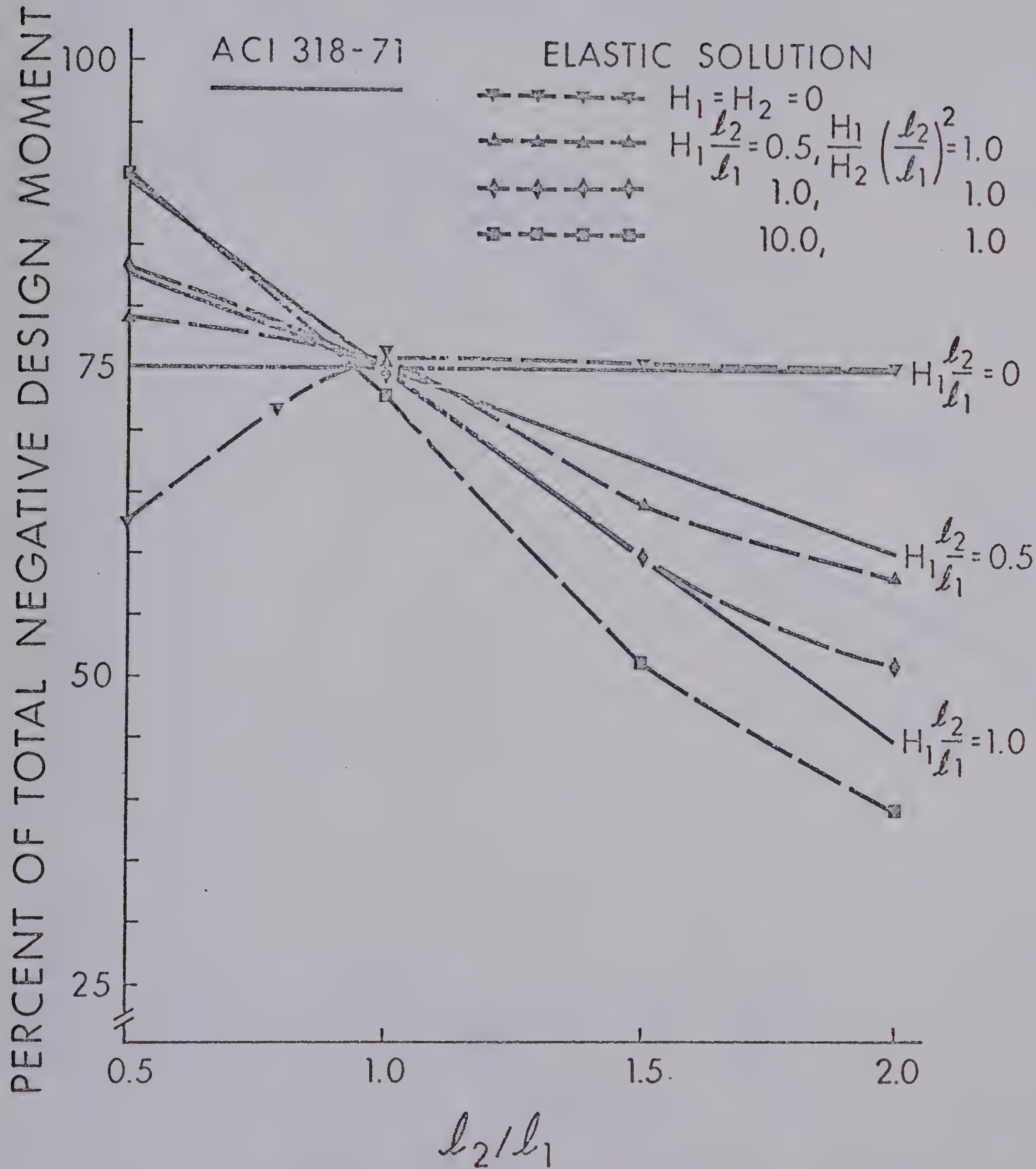


FIGURE 5.13 NEGATIVE COLUMN-STRIP MOMENT, INTERIOR PANEL

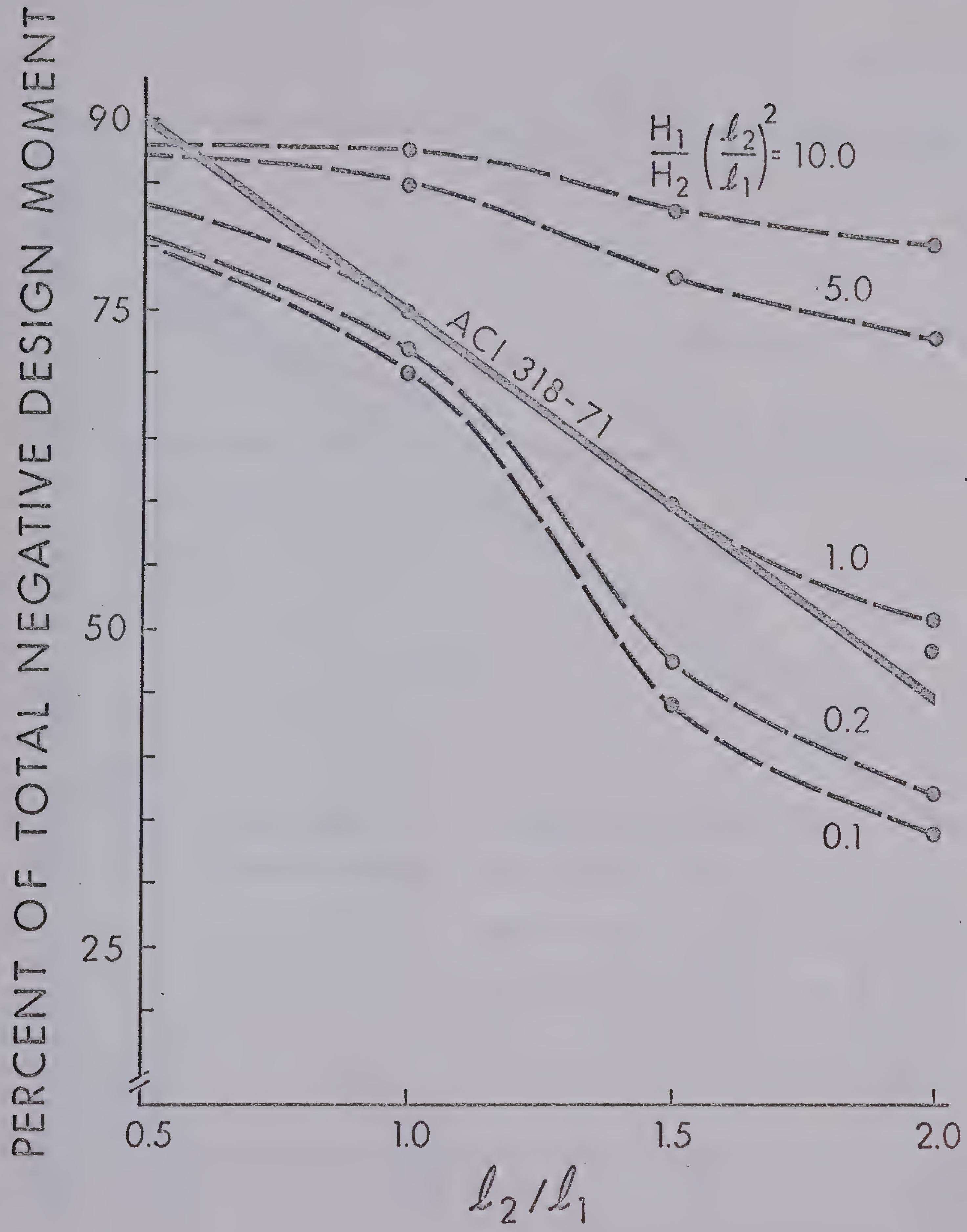


FIGURE 5.14 NEGATIVE COLUMN STRIP MOMENT, INTERIOR PANEL, $H_1\ell_2/\ell_1 = 1$

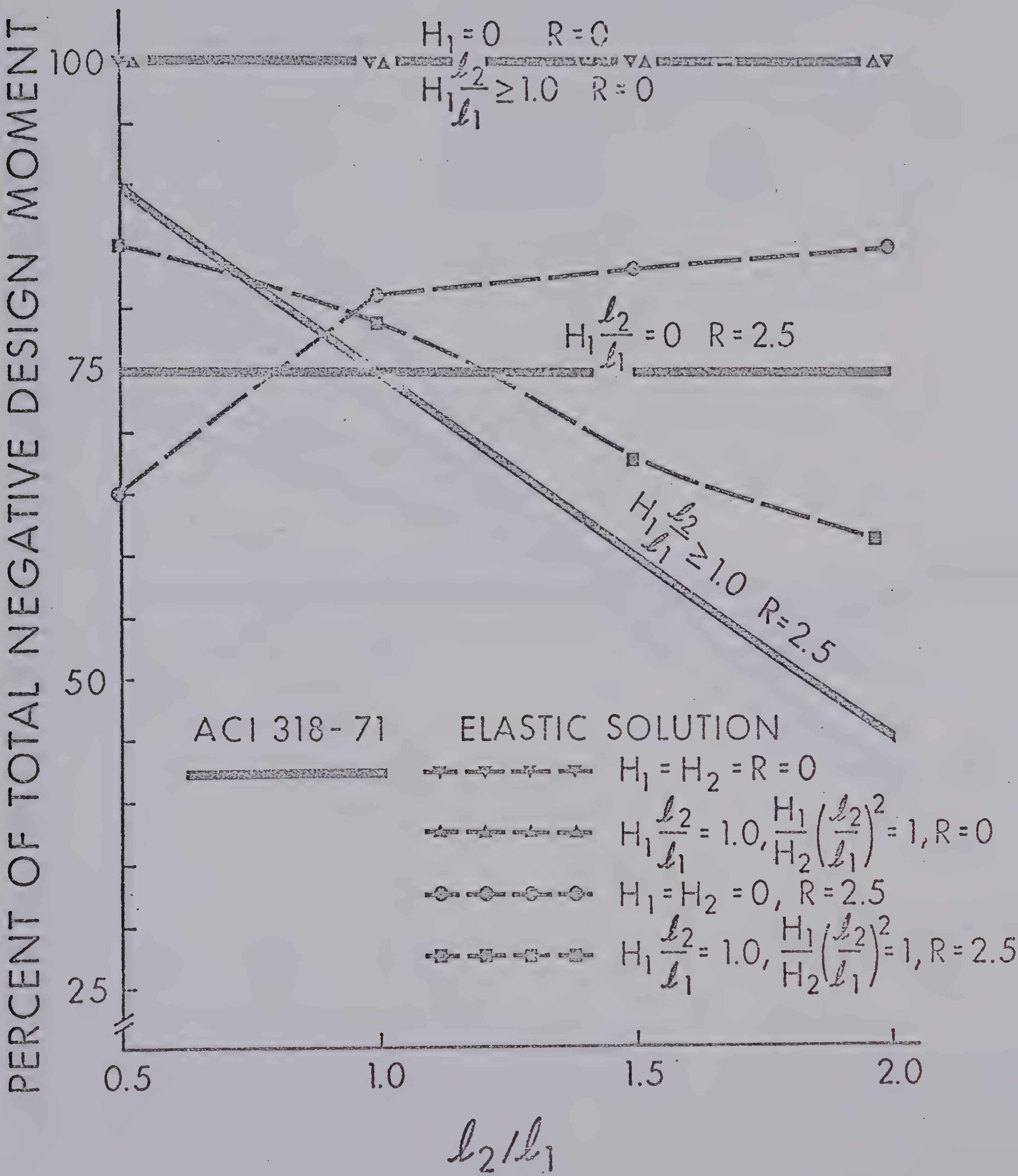


FIGURE 5.15 NEGATIVE COLUMN-STRIP MOMENT, EXTERIOR PANEL

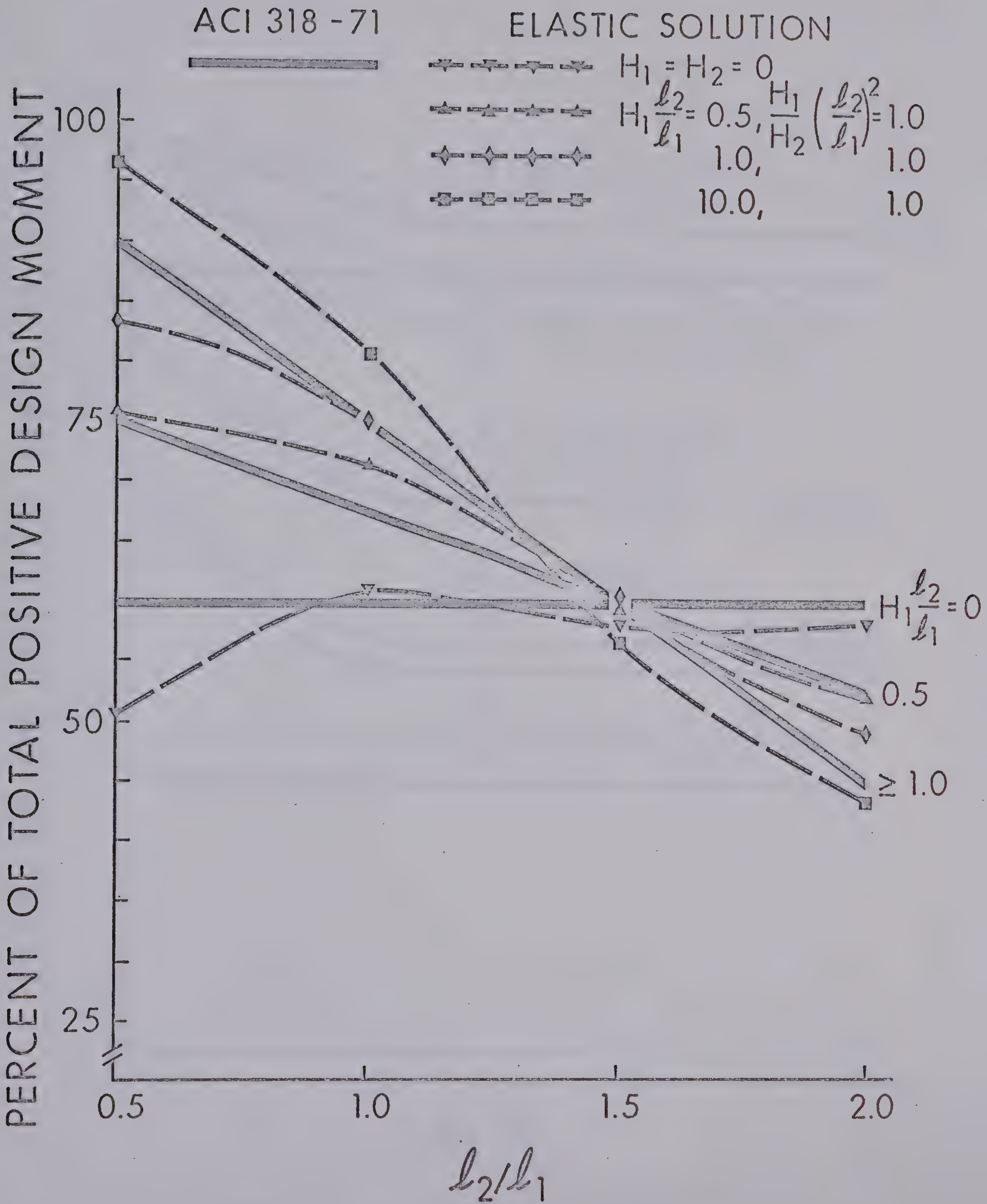


FIGURE 5.16 POSITIVE COLUMN-STRIP MOMENT

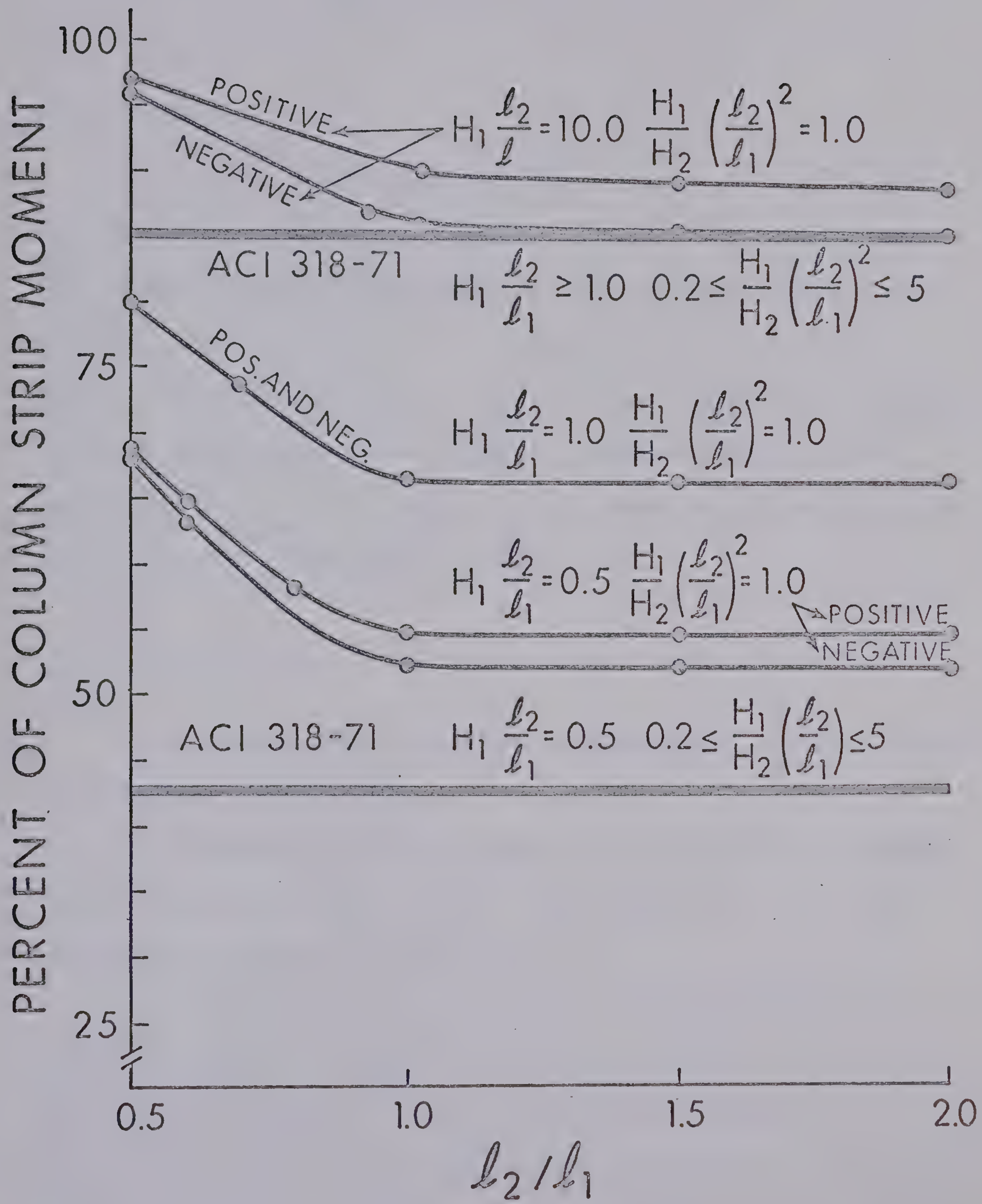


FIGURE 5.17 BEAM DESIGN MOMENT

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary

This study compares the moments in reinforced concrete slabs as determined by proposed ACI 318-71 and an elastic solution based on the finite difference technique. Discrepancies and similarities are pointed out and in certain instances alternate solutions are suggested. A tabular procedure for applying the Equivalent Frame Analysis which can be easily programmed or solved using a desk calculator has been developed

6.2 Conclusions

On the basis of this investigation the following conclusions can be made:

1) Extremely good accuracy is obtained by the elastic solution based on the static check and comparisons with tabulated values by Timoshenko[6].

2) For all regions except the exterior negative moment region, the agreement between elastic solutions and the values proposed by the Direct Design Method is good for square panels. In general the agreement is less good for rectangular panels although still satisfactory for design purposes

3. The exterior negative design moment by the Direct Design Method differs substantially from values obtained by elastic analysis, and an alternative procedure for determining this moment is proposed.

4. The proportion of design moment assigned to the column strip by the procedure in the Code is in good agreement with the elastic analysis when the ratio $H_1 \ell_2^2 / H \ell_1^2$ is equal to unity. For the lower limit of this ratio proposed by the Code, namely 0.2, the agreement is still satisfactory. However, the Code substantially underestimates the column strip moment for the upper limit of 5.0. It is suggested that this limit be reduced to 2.0

5. The limitations given in section 13.3.1.6 for the Direct Design Method should also apply to the Equivalent Frame Analysis.

6. To extend the range of applicability of the procedures given in the Code for assigning moment to column and middle strips, formulas are presented which also give better agreement with elastic solutions.

7. The portion of the column strip moment assigned to the beam by the Code differs substantially from the elastic solution values for all ranges of variables due to over simplification of the procedure. An expression is presented for computing the proportion assigned to the beam which gives excellent agreement with the elastic solutions.

8. The proposed modifications to the Direct Design Method labeled "Suggested Procedure" in this study not only give results substantially closer to those of the elastic solution or on the safe side when expected re-distribution occurs, but also are more general in application in that they are not restricted by limitations given in Code section 13.3.1.6.

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APPENDIX A

EQUIVALENT FRAME ANALYSIS

A.1 Notation*

$k_{j\text{-}col}$	=	sum of the stiffnesses of column above and below slab at joint j.
$k_{j\text{-}i}$, $k_{j\text{-}k}$	=	flexural stiffness of the member j-i and j-k, respectively.
$k'_{j\text{-}i}$, $k'_{j\text{-}k}$	=	product of multiplication of the stiffness and carry-over factors for the member j-i and j-k, respectively.
K_i, K_j, K_k	=	sum of flexural stiffnesses of all members meeting at joint i, j and k, respectively.
m	=	c_1/ℓ_1, ratio of column length to span length in direction moments are being considered.
n	=	c_2/ℓ_2, ratio of column width to span length transverse to the direction moment are being considered.

* For sake of clarity the notation for this Appendix is presented separately from the general nomenclature. Notation used in this appendix is based on Fig. A-1.

M_{j-col} = sum of the column bending moments above and below slab at joint j.

$\overline{M}_i$, $\overline{M}_j$, $\overline{M}_k$ = sum of the fixed-end moments in i, j and k, respectively.

M_{j-i}, M_{j-k} = bending moments at the j-th joint of the member j-i and j-k, respectively.

$\overline{M}_{j-i}$, $\overline{M}_{j-k}$ = fixed-end moments at the j-th joint of the member j-i and j-k, respectively.

ϕ_i, ϕ_j, ϕ_k = rotation, in radians, of the joints i, j, and k, respectively.

A.2 Derivation of General Expressions for Bending Bending Moments in Equivalent Frame

According to section 13.4.1.2 of proposed ACI 318-71 a structure composed of slabs, beams and columns may be analysed as an equivalent frame. This frame is assumed to have columns fixed at their remote ends, and for determining bending moments at a given support, slab-beams which are fixed at any support two panels distant therefrom, provided that the slab continues beyond that joint.

For a given geometric properties of the frame and loading conditions, in order to solve for bending moments M_{j-i} and M_{j-k} at joint j (Fig. A.1) it would be necessary to know rotations of the joints i, j, and k. Using slope deflection principles, it is possible to write three equations of equilibrium with three unknowns, ϕ_i ϕ_j and ϕ_k, as follows:

$$K_i \phi_i + k'_{j-i} \phi_j + \overline{M}_i = 0$$

$$k'_{i-j} \phi_j + K_j \phi_j + k'_{k-j} \phi_k + \overline{M}_j = 0 \qquad \text{(A.1)}$$

$$k'_{j-k} \phi_j + K_k \phi_k + \overline{M}_k = 0$$

At joint j, general equation for the bending moments will be

$$M_{j-i} = \overline{M}_{j-i} + k_{j-i}\phi_j + k'_{j-i}\phi_i$$

$$M_{j-k} = \overline{M}_{j-k} + k_{j-k}\phi_j + k'_{j-k}\phi_k \qquad (A.2)$$

$$M_{j-col} = k_{col}\phi_j$$

Solving equations (A.1) for ϕ_i, ϕ_j and ϕ_k and substituting these into equations (A.2) the expressions for the bending moments become:

$$M_{j-i} = \overline{M}_{j-i} + \phi_j \nu_{j-i} - \frac{k'_{j-i}}{K_i} \overline{M}_i$$

$$M_{j-k} = \overline{M}_{j-k} + \phi_j \nu_{j-k} - \frac{k'_{j-k}}{K_k} \overline{M}_k \qquad (A.3)$$

$$M_{j-col} = k_{j-col}\phi_j$$

where

$$\phi_j = \frac{K_k k'_{j-i} \overline{M}_i - K_i K_k \overline{M}_j + K_i k'_{j-k} \overline{M}_k}{K_i K_j K_k - K_i (k'_{j-k})^2 - K_k (k'_{j-i})^2}$$

$$\nu_{j-i} = k_{j-i} - \frac{(k'_{j-i})^2}{K_i} \tag{A.4}$$

$$\nu_{j-k} = k_{j-k} - \frac{(k'_{j-i})^2}{K_k}$$

The rotation ϕ_j, for two exterior joints (in following example joints 1 and 8) becomes indeterminate and is found by L'Hopital's rule. Solutions are:

for the extreme left joint,

$$\phi_j = \frac{-K_k \overline{M_j} + k'_{j-k} \overline{M_k}}{K_j K_k - (k'_{j-k})^2} \tag{A.5}$$

and for the extreme right joint,

$$\phi_j = \frac{k'_{j-i} \overline{M_i} - K_i \overline{M_j}}{K_i K_j - (k'_{j-i})^2} \tag{A.6}$$

In the above equations, the coefficients for the flexural stiffness and for fixed-end moments must account for variations in the moment of inertia of the slab-beam members and columns along their axes. Examples of these coefficients are given in Tables 3.1 to 3.4 for rectangular columns and in Ref. 13 for when the ratios of

column length to span length in two perpendicular directions are equal. The complete procedure which is most suitable for use with hand calculators can be outlined as follows:

1) For given equivalent frame dimensions and loading conditions calculate stiffnesses and fixed-end moments using Tables 3.1 to 3.4.
2) Calculate factors ϕ and ν using formulas (A.4) or (A.5) and (A.6) for exterior joints.
3) From formulas (A.3) obtain the beam and column bending moments.
4) Check for joint equilibrium.
5) Reduce negative moments to those at column face.

A.3 Numerical Example

In order to illustrate above procedure a seven span equivalent frame (Fig. A.2) has been analysed in Table A.1. Three loading conditions were assumed: all panel loading, odd panel loading and even panel loading. It should be noted that only one panel has been loaded at a time for the second and third loading condition. Using the procedure derived in section A.2 it is possible to obtain bending moments corresponding to a unit loading. These are listed in columns 19, 20 and 21 of Table A.1. Combining these "unit" moments for design dead load and

design live load, negative moments were obtained for the all panel loading, adjacent panel loading and single panel loading. These moments are listed in columns 22, 23 and 24. The critical negative moments are selected from columns 22 and 23 of Table A.1 and the critical positive moments from columns 22 and 24. Negative moments are based on spans measured center to center of columns. According to sections 13.4.2. of ACI 318-71 these can be reduced to the face of the columns. Corley's[1] expression was used for the reduction in this example.

TABLE A.1 EQUIVALENT FRAME ANALYSIS PROCEDURE +

Joint	Section	n=0.1 m	k'	k	K	$\bar{M}$	$\Sigma\bar{M}$	ϕ lc1**	ϕ lc2	ϕ lc3	ν	$\phi\ \nu$ lc.1	$\phi\ \nu$ lc.2	$\phi\ \nu$ lc.3	$-\frac{k'_{j-i}}{K_i}\Sigma\bar{M}i$ or $-\frac{k'_{j-k}}{K_k}\Sigma\bar{M}_k$ lc.1	lc.2	lc.3
1	2	3	4	5	6	7	8	9	10	11	12	13 = 12 x9	14 = 12x10	15 = 12x11	16	17	18
1	1-2 * 1-col	0.200	0.0544	0.1040 1.0895	1.1935	-12.384	-12.384	10.6271	10.8125		0.1018	1.082	1.101		-0.276	-0.497	
2	2-1 2-3 * 2-col	0.200 0.300	0.0544 0.0852	0.1040 0.1616 1.0895	1.3551	12.384 -5.504	6.880	-6.5829	-9.6120	4.3497	0.1015 0.1561	-0.668 -1.028	-0.976 1.500	0.443 0.679	0.564 1.424	0.564	-0.396
3	3-2 3-4 * 3-Col	0.300 0.134	0.0852 0.0335	0.1616 0.0678 1.0895	1.3189	5.504 -27.540	-22.036	16.6651	21.5625	4.4568	0.1562 0.0669	2.602 1.115	3.368 1.443	-0.696 -0.298	-0.433 0.594	-0.763	0.346
4	4-3 4-5 * 4-col	0.134 0.100	0.0335 0.0257	0.0678 0.0512 1.0895	1.2085	27.540 -48.960	-21.420	17.9911	-23.3936	41.3588	0.0670 0.0507	1.205 0.012	-1.566 -1.186	2.765 2.097	0.559 -0.857	0.699	0.966
5	5-4 5-6 * 5-col	0.100 0.300	0.0257 0.0852	0.0512 0.1616 0.0895	1.3023	48.960 -5.504	43.456	-34.5453	4.5172	38.5720	0.0507 0.1562	-1.751 -5.396	0.229 0.706	1.956 6.025	0.455 0.870	-0.352	1.041
6	6-5 6-7 * 6-col	0.300 0.160	0.0852 0.0422	0.1616 0.0822 1.0895	1.3333	5.504 -19.125	-13.621	12.6707	-4.4212	14.8877	0.1560 0.0808	1.977 1.024	0.690 0.357	2.322 1.203	2.843 0.342	0.360	0.621
7	7-6 7-8 * 7-col	0.160 0.240	0.0422 0.0665	0.0822 0.1269 1.0895	1.2986	19.125 8.600	10.525	-8.1058	7.0114	-15.2518	0.0809 0.1233	-0.656 -0.999	0.567 0.865	-1.234 -1.881	0.431 -0.470	-0.470	0.608
8	8-7 * 8-col	0.240	0.0655	0.1269 1.0895	1.2164	8.600	8.600	-6.6456	7.4530		0.1235	-0.821	-0.920	-0.539	0.440		

** Load condition

\+ See Fig. A.2

* Refer to summation of stiffness or bending moments of columns above and below corresponding joint.

TABLE A.1 (continued)

EQUIVALENT FRAME ANALYSIS PROCEDURE

JOINT	$M_{(unit\ load)}$			M^{***} (actual load)			M^{***} (design)	
	lc.1	lc. 2	lc. 3	all panel loading	adjacent panel loading	skip panel loading	NEGATIVE	POSITIVE
	19 = 7+13+16	20 = 7+14+17	21 = 7+15+18	22 = $19x(W_D+W_L)$	23 = $19x(W_D+$ $(20+21)W_L$	24 = $19W_D+20\ W_L$ or $19W_D+21W_L$	25	26
1	-11.578 11.578	-11.780 11.780		-57.43 57.43	- 49.97 49.97	-49.97 49.97	-30.91	
								26.09
2	12.280 -5.108 -7.172	11.972 -1.500 -10.472	0.443 -5.181 4.739	60.91 -25.34 - 35.57	52.83 -25.07 - 27.76	51.93 -22.01 37.43	34.39 -7.75	
								9.39
3	7.673 -25.831 18.157	3.368 -26.860 23.492	5.154 -0.298 -4.856	38.06 - 128.12 90.06	34.57 - 113.26 78.69	27.70 -112.66 88.60	20.47 -88.21	
								56.99
4	29.304 - 48.905 19.601	26.673 - 1.186 -25.487	2.769 -47.829 45.060	145.35 - 242.57 97.22	125.70 - 209.54 83.84	120.05 -207.12 135.83	105.44 -189.27	
								102.21
5	47.664 -10.030 37.636	0.229 - 5.150 4.922	48.046 - 6.024 -42.023	236.41 -49.75 -186.67	205.25 45.26 -159.99	204.78 32.97 170.03	183.11 32.16	
								7.29
6	4.638 -18.443 13.805	5.174 -0.357 -4.817	2.322 18.543 16.220	23.00 91.48 68.47	25.68 -79.87 54.19	20.94 79.14 64.01	10.51 58.26	
								40.77
7	18.900 -10.069 - 8.831	0.567 - 8.205 . 7.639	18.496 -1.881 -16.617	93.74 - 49.94 - 43.80	81.22 -43.13 -38.09	80.07 -39.29 53.68	60.52 -27.88	
								17.47
8	7.240 - 7.240	8.120 -8.120		35.91 -35.91	32.78 -32.78	32.78 -32.78	13.85	

*** in ft-kips

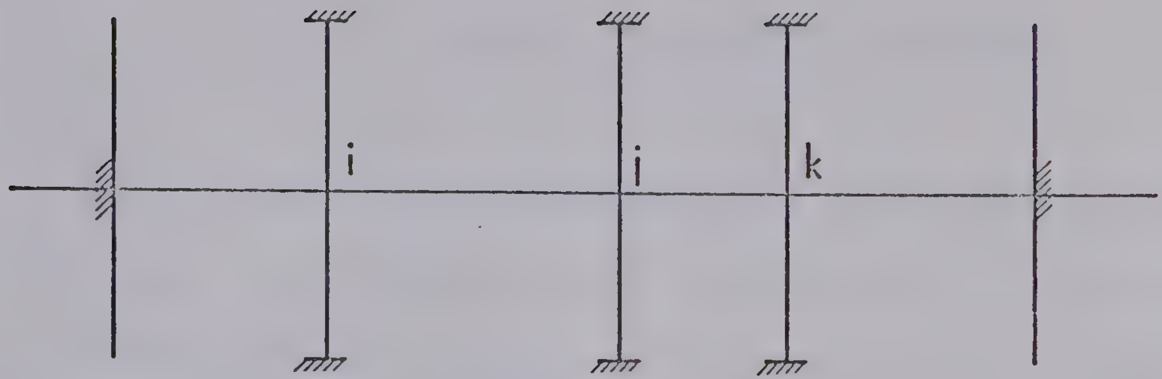
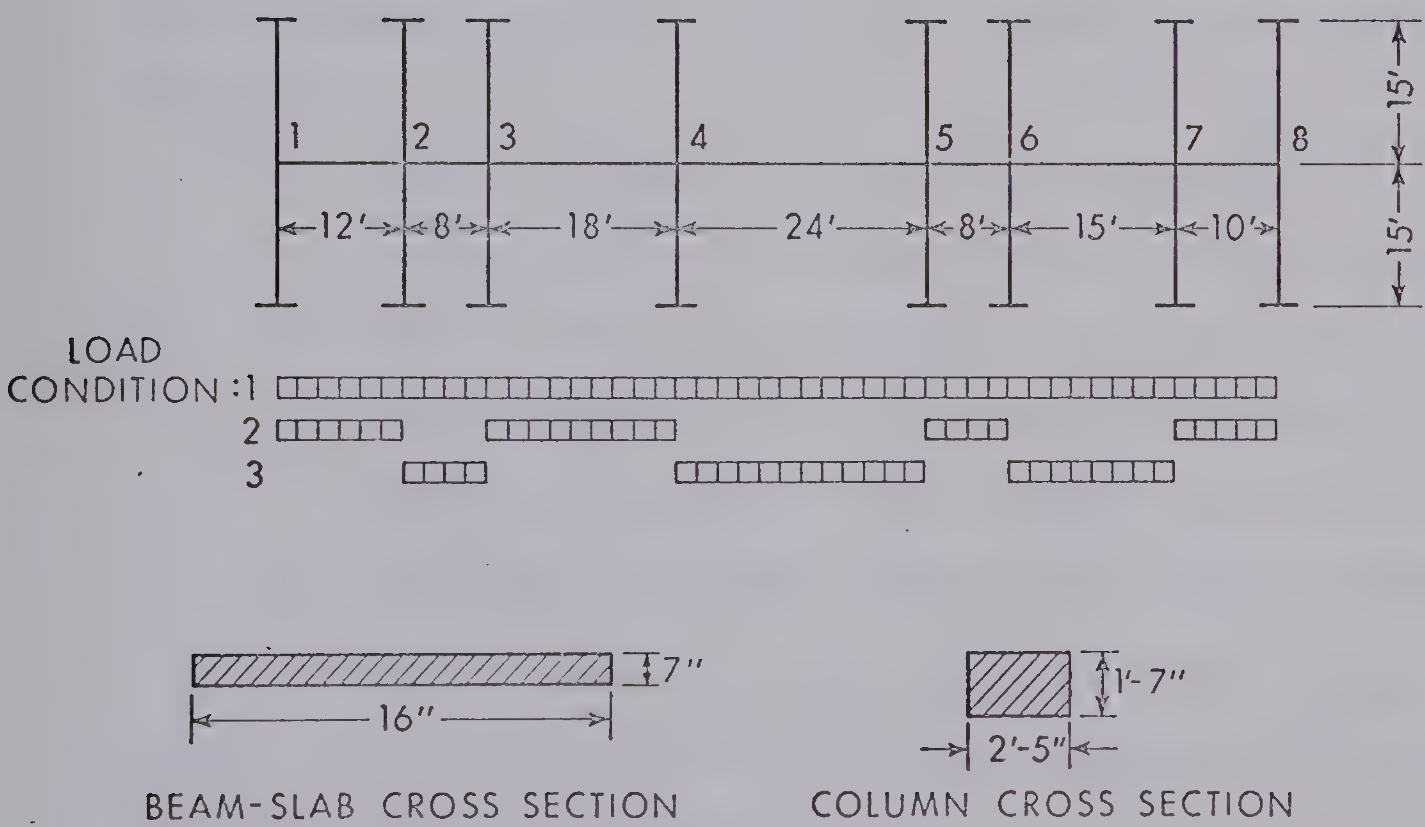


FIGURE A.1 GENERAL POSITION OF EQUIVALENT FRAME



LOADING : $W_D = W_L = 0.1\ k/ft^2 = 1.6\ k/ft$

FIGURE A.2 AN EXAMPLE OF EQUIVALENT FRAME

APPENDIX B

FINITE DIFFERENCE METHOD

The basic assumptions for the ordinary theory of plates and beams can be found in Ref. 6. Assumptions involved in the concept of Newmark's plate analog are listed in Ref. 8 and assumptions required to count for the effects of beams are listed in Ref. 2. However some additional assumptions are required for the slab supported with elongated columns, $c_1/\ell_1 = 0.2$. These can be stated as follows:

a) Column stiffness in direction transverse to elongation is considered concentrated along axis.

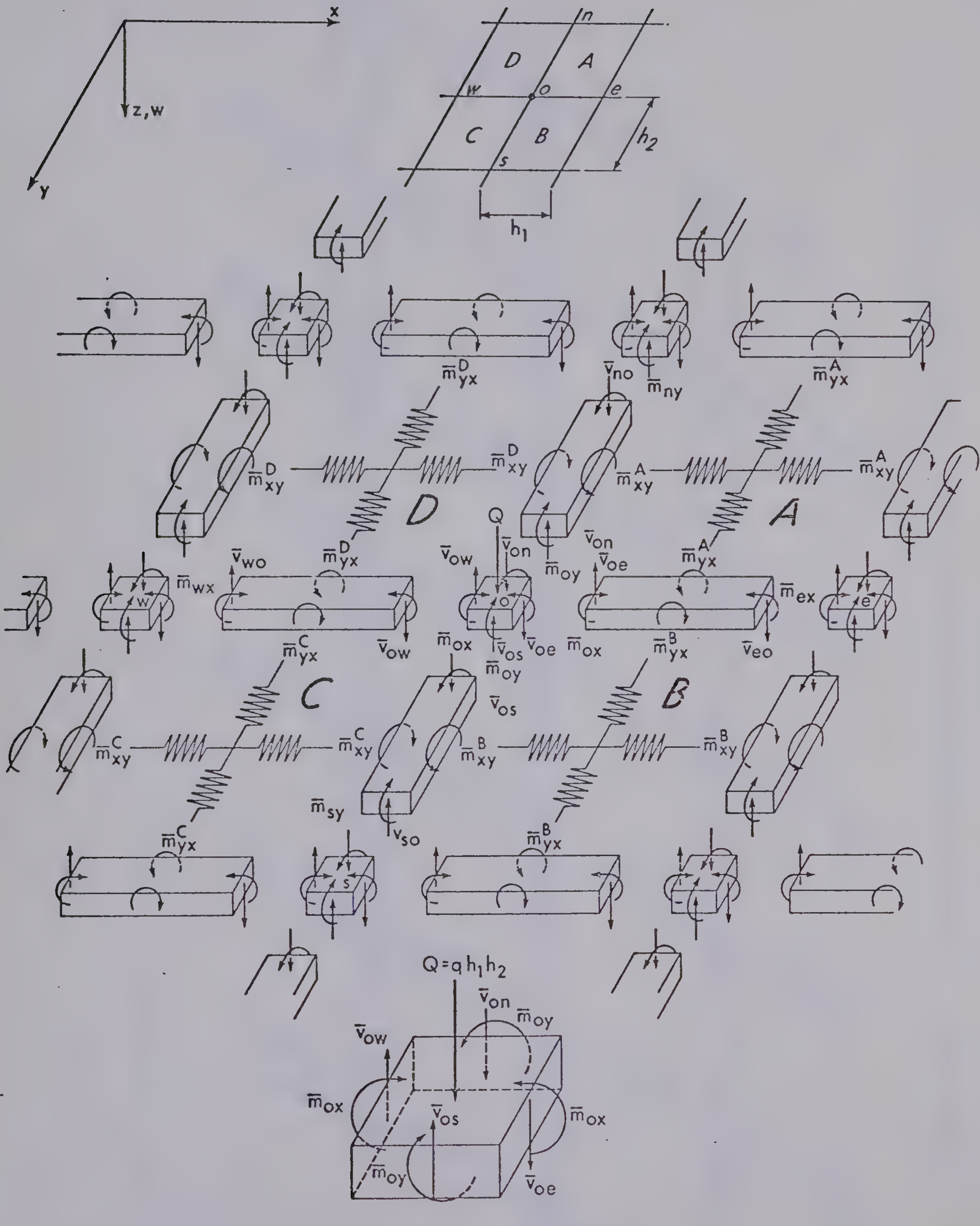
b) Column cross-section remains as a plane ("line") after deformations. This leads to linearly related deformations along column cross-section.

c) Rotations of column occurs about column centerline.

For the complete solution of slab described in section 3.4.2, forty finite difference operators, based on Newmark plate analog were derived.

Figs. B.1 and B.2 represent system of forces in a plate analog affecting equilibrium of a typical joint and interior column, respectively. In Figs. B.3 and B.4 finite difference operators are given for a typical plate joint

and for the plate-column joint one grid spacing removed from column centerline. It should be noted that R was used for designation of rectangularity of finite difference elements (h_1/h_2) in this appendix only.



Enlarged Joint o

FIGURE B.1 FORCES AFFECTING EQUILIBRIUM OF TYPICAL JOINT IN PLATE ANALOG

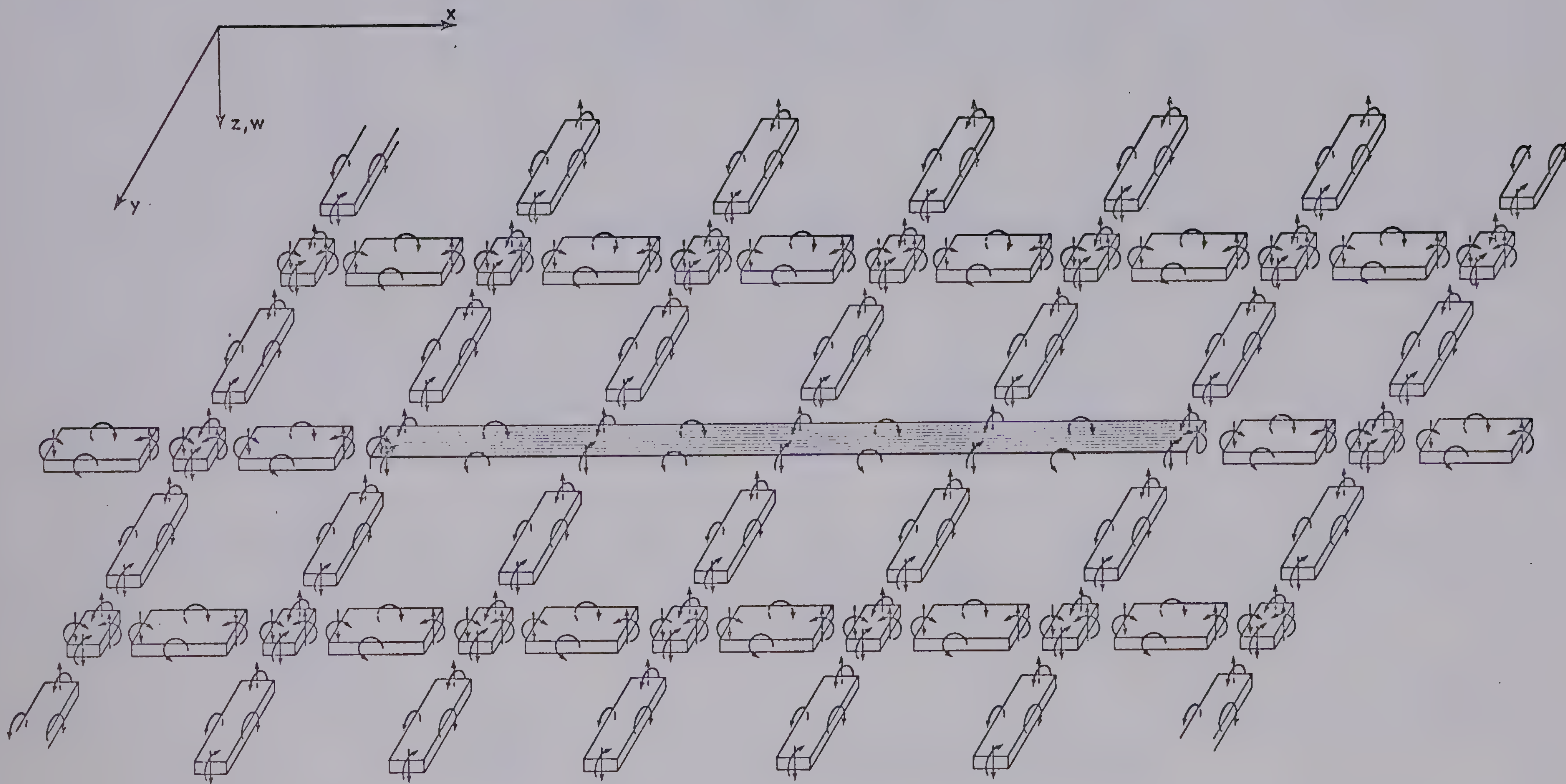


FIGURE B.2 FORCES AFFECTING EQUILIBRIUM OF INTERIOR COLUMN

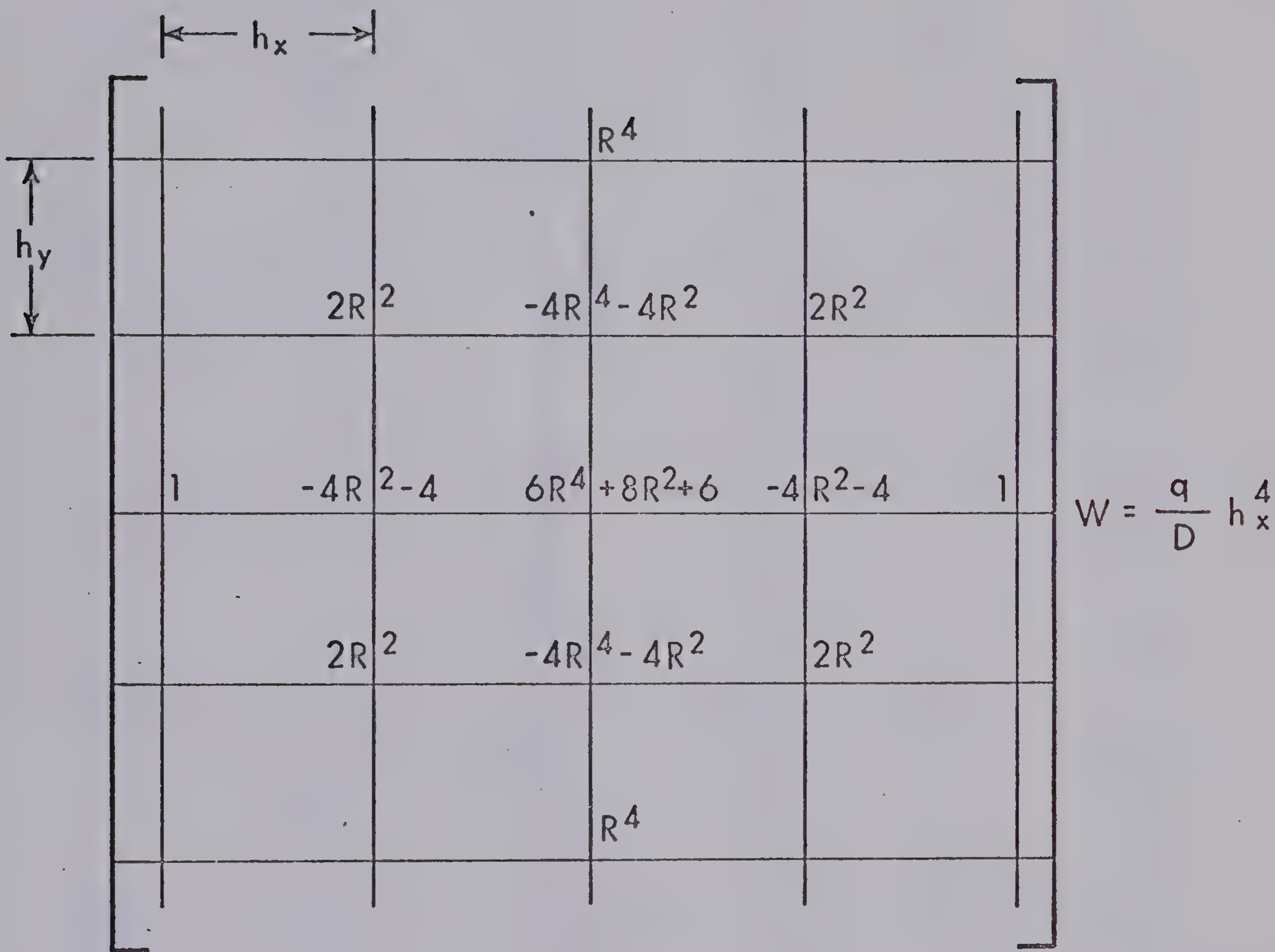


FIGURE B.3 FINITE DIFFERENCE PATTERN FOR GENERAL PLATE JOINT

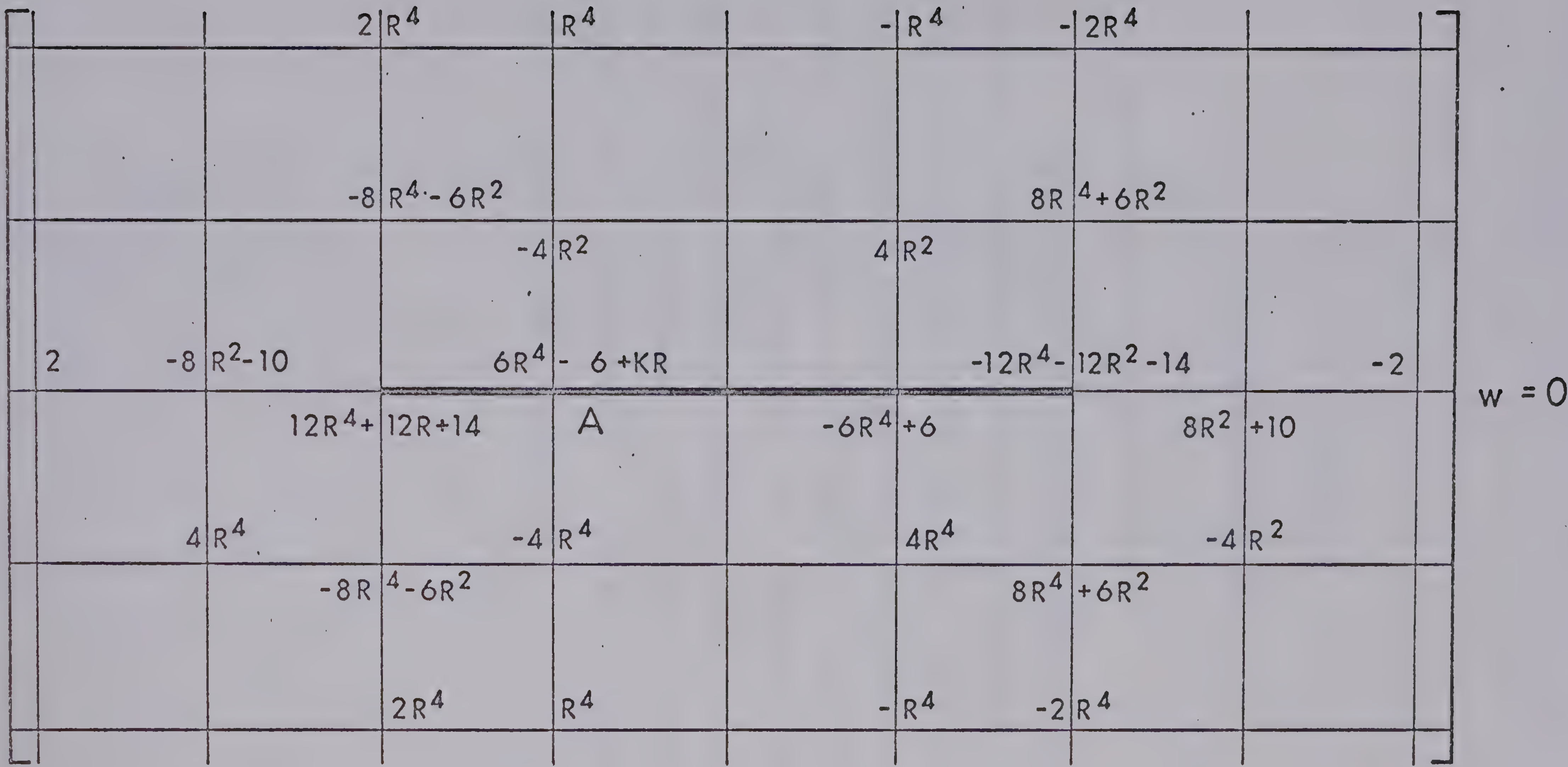


FIGURE B.4 FINITE DIFFERENCE OPERATOR FOR THE PLATE-COLUMN JOINT A, ONE GRID SIZE LEFT OF THE COLUMN CENTERLINE.

APPENDIX C

COMPUTER PROGRAM

C.1 Description of Computer Program

In order to establish the operator coefficients and solve the resulting simultaneous equations for deflections, moments and shears, forty four subroutines were employed in conjunction with the main line program. Because of complexity of the computer program and the large number of statements (1950 statements) detailed flow charts and listings are not included in this thesis but can be obtained from the Department of Civil Engineering, The University of Alberta. A general flow chart is shown in next section.

C.2 General Flow Diagram